

Dynamical D-Terms in Supergravity

Valerie Domcke^a, Kai Schmitz^b, Tsutomu T. Yanagida^b

^a *SISSA/INFN, 34100 Trieste, Italy*

^b *Kavli IPMU (WPI), University of Tokyo, Kashiwa 277-8583, Japan*

Abstract

Most phenomenological models of supersymmetry breaking rely on nonzero F-terms rather than nonzero D-terms. An important reason why D-terms are often neglected is that it turns out to be very challenging to realize D-terms at energies parametrically smaller than the Planck scale in supergravity. As we demonstrate in this paper, all conventional difficulties may, however, be overcome if the generation of the D-term is based on strong dynamics. To illustrate our idea, we focus on a certain class of vector-like SUSY breaking models that enjoy a minimal particle content and which may be easily embedded into more complete scenarios. We are then able to show that, upon gauging a global flavor symmetry, an appropriate choice of Yukawa couplings readily allows to dynamically generate a D-term at an almost arbitrary energy scale. This includes in particular the natural and consistent realization of D-terms around, above and below the scale of grand unification in supergravity, without the need for fine-tuning of any model parameters. Our construction might therefore bear the potential to open up a new direction for model building in supersymmetry and early universe cosmology.

Contents

1	Introduction and preliminaries	2
1.1	Constant field-independent FI-terms in supergravity	2
1.2	Effective field-dependent FI-terms from string theory	3
1.3	Dynamical FI-terms in strongly interacting gauge theories	5
2	Minimal setup based on $SP(1)$ dynamics ($\cong SU(2)$ dynamics)	7
2.1	The IYIT model of dynamical supersymmetry breaking	7
2.2	Effective FI-term upon weakly gauging a global $U(1)$ flavor symmetry	9
2.3	Consistent embedding into supergravity	13
2.4	Mass spectrum and stabilization of the sgoldstino direction	15
3	Generalizations	21
3.1	Dynamical D-terms based on $SP(N_c)$ dynamics	21
3.2	Dynamical D-terms based on $SU(N_c)$ dynamics	24
4	Applications	26
4.1	Mediation of supersymmetry breaking to the visible sector	27
4.2	Cosmic inflation driven by a dynamical D-term	28
5	Conclusions and outlook	30

1 Introduction and preliminaries

In this paper, we wish to illustrate how an effective Fayet-Iliopoulos (FI) D-term may be dynamically generated at an intermediate energy scale in strongly interacting supersymmetric gauge theories. In Secs. 1.1 and 1.2, we shall first review the well-known problems related to the existing constructions of constant and effective FI-terms in the literature. The reader acquainted with these issues may therefore directly skip to Sec. 1.3, in which we outline our basic idea.

1.1 Constant field-independent FI-terms in supergravity

In any realistic supersymmetric extension of the standard model, supersymmetry (SUSY) needs to be spontaneously broken in some hidden sector. The order parameters of spontaneous SUSY breaking in a given supersymmetric theory are the expectation values of the auxiliary F and D fields. While models that break SUSY via nonzero F-terms are referred to as O’Raifeartaigh models [1], models based on nonzero D-terms always feature a realization of the Fayet-Iliopoulos mechanism [2], which is why they are also known as FI models of SUSY breaking. The crucial observation behind SUSY breaking via (Abelian) D-terms is that the Lagrangian \mathcal{L} of a $U(1)$

gauge theory also admits the following supersymmetric and gauge-invariant operator,

$$\mathcal{L}_{\text{FI}} = \int d^4\theta K_{\text{FI}} = -g\xi D, \quad K_{\text{FI}} = -2g\xi V, \quad (1)$$

where K_{FI} is part of the Kähler potential, $V \sim (\lambda, A, D)$ represents the vector superfield containing the $U(1)$ gauge degrees of freedom (DOFs), ξ is a free parameter of mass-dimension 2, g stands for the $U(1)$ gauge coupling constant, and θ denotes the anticommuting superspace coordinate. If one manages to stabilize all scalars carrying nonzero $U(1)$ gauge charge around their origin, the operator in Eq. (1) leads to a nonvanishing D-term scalar potential belonging to the $U(1)$ gauge interactions, $V_D \propto g^2\xi^2$, and hence to the spontaneous breaking of SUSY. This mechanism has several interesting phenomenological applications in supersymmetric model building as well as in cosmology. A nonvanishing FI-term \mathcal{L}_{FI} can, for instance, play a crucial role in mediating SUSY breaking to the visible sector or provide the vacuum energy density that is necessary to drive the inflationary stage in the very early universe [3].

Despite their abundant occurrence in the literature on SUSY phenomenology over the last four decades, some important aspects of FI-terms have, however, become clear only in the past few years [4–6]. As it turns out, it is in fact very difficult or even impossible to consistently couple a $U(1)$ gauge theory featuring a genuine (i.e. constant, field-independent) FI-term to minimal supergravity (SUGRA). If the FI parameter ξ is assumed to be a fundamental constant, coupling the rigid theory to gravity requires that the final locally supersymmetric theory must have an additional exact global continuous symmetry.¹ As shown in Refs. [4, 6], this result is independent of the SUGRA formalism and equally applies in the old [9] as well as in the new minimal [10] off-shell formulation of SUGRA. According to general rules of quantum gravity, all global symmetries are, however, eventually broken by gravity effects [11].² A theory based on minimal SUGRA and exhibiting a global symmetry is therefore necessarily inconsistent. This also explains why all attempts to find string compactifications with genuine FI-terms in the low-energy effective theory have been futile so far.

1.2 Effective field-dependent FI-terms from string theory

A possible way out of these difficulties is to resort to field-dependent FI-terms, in the case of which ξ is regarded as an effective parameter that actually depends on the vacuum expectation

¹This conclusion can be avoided if the rigid theory only contains fields with vanishing $U(1)$ charge, cf. Ref. [7] for an explicit model, or if $\xi/2$ is quantized in units of the reduced Planck mass $M_{\text{Pl}} = (8\pi G)^{-1/2}$ [8]. The latter is always the case once the underlying $U(1)$ gauge group is assumed to be compact, i.e. when its global topology is that of a true $U(1)$ and not the one of the real numbers \mathbb{R} . For noncompact global topology, ξ can be parametrically small, $\xi/M_{\text{Pl}}^2 \ll 1$; but then the SUGRA theory needs to exhibit a global continuous symmetry.

²Even if one disregards this conceptional argument about the general properties of quantum gravity, theories with a constant FI-term are still in trouble for phenomenological reasons. Upon the coupling to SUGRA, the initial non- R $U(1)$ gauge symmetry turns into a continuous local R symmetry. In the context of the standard model, every such symmetry is, however, necessarily anomalous, which renders the entire theory inconsistent at the quantum level. We are thankful to W. Buchmüller and R. Kallosh for a helpful discussion on this point.

values (VEVs) of other scalar fields, $\xi = \xi(\langle\phi_i\rangle)$. The generation of such field-dependent FI-terms is therefore always associated with the spontaneous breaking of the $U(1)$ gauge symmetry and one should actually not refer to them as FI-terms. Instead, they merely correspond to the VEV of the auxiliary D field in the new vacuum after spontaneous symmetry breaking, $\xi \equiv \langle D \rangle / g$. As pointed in Ref. [4], the fundamental obstacle in coupling a theory with a constant FI-term to SUGRA, which eventually also necessitates the introduction of a global symmetry, is the fact that such theories do not possess a gauge-invariant Ferrara-Zumino (FZ) supercurrent multiplet [12]. By appropriately choosing the gauge transformation behavior of the fields ϕ_i , the gauge invariance of the FZ-multiplet can, however, be preserved and the theory can be consistently coupled to SUGRA within the old minimal formalism. A famous example of such a construction are the field-dependent FI-terms frequently encountered in string theory [13], which are based on the Green-Schwarz mechanism [14] of anomaly cancellation,

$$K_{\text{FI}} = f_{\text{GS}}(V + \Phi + \Phi^\dagger). \quad (2)$$

Here, f_{GS} is an appropriate function of the linear combination $V + \Phi + \Phi^\dagger$ and Φ stands for a (not necessarily properly normalized) modulus field that transforms in the affine representation of the (noncompact) $U(1)$ gauge group. An alternative approach to deal with the non-gauge invariance of the FZ-multiplet in the presence of a constant FI-term is to trade the FZ-multiplet for the so-called \mathcal{S} -multiplet [6], which is always well-defined. Gauging the \mathcal{S} -multiplet rather than the FZ-multiplet then amounts to coupling the rigid theory to 16/16 SUGRA [15] rather than to minimal SUGRA. In this non-minimal framework for SUGRA, the gravity multiplet contains an additional chiral matter multiplet next to the ordinary graviton and the ordinary gravitino. Interestingly enough, this additional chiral field can be identified with the above modulus field Φ and the FI-term in the gauged theory ends up being of the same form as in Eq. (2).

Now one, however, faces the problem that the modulus Φ needs to be stabilized at sufficiently high energies, since it would otherwise absorb the effective FI-term in its VEV. This requirement imposes strong constraints on the underlying high-energy theory, which may be hard to fulfill. One possibility in this context could potentially be to rely on a large gravity-mediated mass m_ϕ for the modulus field. Depending on the size of the effective FI parameter ξ_{GS} , this would, however, require an extremely large gravitino mass, $m_\phi \sim m_{3/2} \gtrsim g\sqrt{|\xi_{\text{GS}}|}$ [16]. Alternatively, one may attempt to stabilize the modulus above the SUSY breaking scale by means of a dedicated mechanism. In this case, the vector multiplet V will, however, acquire the same mass as the modulus Φ via the Stückelberg mechanism. Then, once we integrate out the modulus at low energies, also the vector multiplet decouples, such that there is no energy range in which we could meaningfully speak of an effective FI-term for the $U(1)$ vector field. Besides this, even more elaborate attempts to stabilize Φ are not guaranteed to be successful. In Ref. [16], it has, for instance, recently been shown that, in the context of ordinary D-term hybrid inflation, all of the standard, straightforward approaches to stabilize the modulus field Φ are bound to fail.

1.3 Dynamical FI-terms in strongly interacting gauge theories

Because of these limitations of the existing constructions of effective FI-terms in the literature, it is desirable to seek alternative mechanisms for the generation of field-dependent FI-terms which are consistent also in the presence of gravity and which, at the same time, do not lead to any problems related to the modulus field Φ . An attractive possibility in this context, which we will further explore in this paper, is to base the generation of the FI-term on the dynamics of strongly interacting supersymmetric gauge theories.³ Here, our main observation is that, in models of dynamical SUSY breaking (DSB), it is possible to generate nonvanishing D-terms by gauging a global $U(1)$ flavor symmetry, followed by adjusting the VEVs of the resulting charged composite fields at low energies by means of appropriate Yukawa interactions.

In this sense, our construction bears some resemblance to the mechanism described in Ref. [18], which also utilizes strong dynamics to generate an effective D-term. Instead of a superpotential suited for dynamical SUSY breaking, this mechanism, however, relies on a runaway superpotential. While our F-term scalar potential exhibits a stable SUSY-breaking vacuum from the very beginning, the corresponding scalar potential analyzed in Ref. [18] therefore initially comes with a supersymmetric vacuum at infinity. After weakly gauging a global symmetry (just as in our case), the runaway directions in the F-term potential are then stabilized by the D-term contributions to the scalar potential. A further crucial difference between our mechanism and the one presented in Ref. [18] is that we focus on simple vector-like gauge theories, while Ref. [18] only discusses a set of chiral models. Our mechanism hence appears to be more minimal and promises to be more easily applicable in the explicit construction of realistic models.

Now, to see how an effective D-term may be generated in a given vector-like DSB model, imagine that the low-energy DOFs of this theory correspond to, for instance, a set of mesons M^i . Further, suppose that the low-energy effective theory contains a global $U(1)$ flavor symmetry, under which the meson fields carry charges q_i . We are then free to gauge this flavor symmetry, which provides us with a $U(1)$ D-term of the following form,

$$D = -g \sum_i q_i |M^i|^2 + \Delta D, \quad \sum_{\text{all}} q = \sum_{\text{all}} q^3 = 0, \quad (3)$$

where ΔD stands for further contributions to D from additional charged fields and where we implicitly assume that at least some of the charges q_i are nonzero. Also, note that the sum of all charges as well as the sum of all charges cubed are required to vanish in order to ensure anomaly-freedom. As an elementary ingredient of our construction, we emphasize that, in the context of dynamical SUSY breaking, all flat directions in moduli space are necessarily lifted. The mesons are thus guaranteed to acquire well-defined and definite VEVs,

$$\langle M^i \rangle = f_i(\lambda_j, g) \Lambda. \quad (4)$$

Here, Λ is the dynamical scale of the strong interactions and the f_i are model-dependent functions of the Yukawa coupling constants λ_j in the theory as well as of the gauge coupling constant g .

³The first model exploiting this possibility to generate a dynamical FI-term has been presented in Ref. [17].

Note that, in contrast to the corresponding scalar VEVs discussed in Ref. [18], our meson VEVs are also still well-behaved in the limit $g \rightarrow 0$. For appropriate functions f_i , it is then straightforward to generate a nonzero D-term proportional to the dynamical scale, $D \sim g \Lambda^2$.

By construction, the such obtained dynamical D-terms can never be the only source of SUSY breaking. Instead, SUSY is always also broken by the strong dynamics responsible for the VEVs of the composite fields at low energies, cf. Eq. (4). This source of SUSY breaking is associated with one or several nonzero F-terms, the magnitude of which exceeds the one of the D-term. This result is consistent with general theorems in SUGRA, which state that generically the dominant contribution to SUSY breaking is provided by F-terms rather than by D-terms [17–19],

$$|D| \lesssim |F|. \quad (5)$$

An obvious advantage of relying on strong dynamics in generating an effective D-term is that the magnitude of the such obtained D-term is controlled by the dynamical scale Λ , so that it can be easily varied over many orders of magnitude,

$$\sqrt{|\xi|} \sim \Lambda, \quad \Lambda_{\min} \lesssim \Lambda \lesssim M_{\text{Pl}}. \quad (6)$$

Here, Λ_{\min} denotes a model-dependent phenomenological lower bound on the dynamical scale, while the Planck mass M_{Pl} represents a model-independent theoretical upper bound. Depending on the details of the coupling between the strongly interacting and the visible sector, we expect Λ_{\min} to be typically of $\mathcal{O}(100)$ TeV. Meanwhile, we point out that values of the dynamical scale exceeding the Planck scale would take us out of the validity range of SUGRA as a low-energy effective description of quantum gravity and hence such large Λ values are not admissible. We also mention that, in the context of a grand unified theory (GUT), a dynamically generated D-term could very well be of the order of the unification scale, $\Lambda_{\text{GUT}} \simeq 2 \times 10^{16}$ GeV. This would certainly be particularly appealing from the perspective of both particle physics and cosmology.

The range of viable Λ values in our dynamical setup, cf. Eq. (6), needs to be contrasted with the expected size of the anomalous FI-term in string theory. All relevant energy scales in string theory, the compactification scale M_c , the string scale M_s as well as the four-dimensional Planck scale M_{Pl} , are all very large, $M_c \sim M_s \sim M_{\text{Pl}} \sim 10^{18}$ GeV, which is why, purely based on dimensional analysis, we would also expect a stringy FI-term to be very large. To make this argument a bit more explicit, suppose that the function f_{GS} in Eq. (2) can be expanded as a Taylor series in $V_M = V + \Phi + \Phi^\dagger$, so that in the vicinity of $V_M = 0$ we are able to write⁴

$$f_{\text{GS}}(V_M) \sim M_{\text{Pl}}^2 \sum_{n=0}^{\infty} \frac{1}{n!} c_n V_M^n, \quad (7)$$

Without any particular fine-tuning among the order $\mathcal{O}(1)$ coefficients c_n , the function f_{GS} is then guaranteed to yield an effective FI-term scale close to the Planck scale, $\sqrt{|\xi_{\text{GS}}|} \sim M_{\text{Pl}}$. The

⁴Note that, in order to realize a non-spurious FI-term along with a kinetic term for the modulus field Φ , this series needs to extend at least up to cubic order in V_M . Otherwise, the linear term, which actually induces the nonvanishing effective D-term, could always be shifted away by a field redefinition.

dynamical generation of an effective D-term in a strongly coupled field theory is hence superior to its stringy alternative based on the Green-Schwarz mechanism in the sense that the former is capable of realizing ξ values in a much larger range than the latter.

In the present paper, we will restrict ourselves to the arguably simplest case and illustrate our idea only by means of DSB models based on $SP(N_c)$ gauge dynamics, i.e. dynamical models breaking SUSY à la IYIT [20]. To this end, we first describe in detail the minimal case of an $SP(1)$ theory⁵ in the following section, before we then comment on the general $SP(N_c)$ case in Sec. 3. Besides this, we also explain in Sec. 3 why DSB models based on $SU(N_c)$ instead of $SP(N_c)$ dynamics fail to provide a basis for the successful generation of an effective D-term. In Sec. 4, we then sketch what kind of effects a dynamically generated D-term may have on the superparticle mass spectrum in the visible sector as well as how it may be used for the construction of inflationary models. Finally, we summarize our results and give an outlook as to how our study may be continued in Sec. 5.

2 Minimal setup based on $SP(1)$ dynamics ($\cong SU(2)$ dynamics)

Our dynamical generation of an effective FI-term will be based on the IYIT model. In Sec. 2.1, we shall first review this model and outline how it accomplishes the dynamical breaking of SUSY via the O’Raifeartaigh mechanism. The reader familiar with this model may directly proceed with Sec. 2.2, in which we explicitly present our construction of the field-dependent FI-term.

2.1 The IYIT model of dynamical supersymmetry breaking

The vector-like model introduced in Ref. [20], sometimes referred to as the IYIT model, represents a minimal example of a supersymmetric gauge theory accomplishing spontaneous SUSY breaking by means of strong dynamics. This model is based on strongly interacting $SP(N_c)$ gauge dynamics and features $2N_f = 2(N_c + 1)$ chiral quark (i.e. matter) fields Q^i transforming in the fundamental representation of $SP(N_c)$. At energies below the dynamical scale Λ , the interaction between these quark fields is best described in terms of the $2N_f(2N_f - 1)/2$ gauge-invariant composite meson fields $M^{ij} = -M^{ji} = Q^i Q^j / \Lambda$. Compared to other DSB models, the field content of the low-energy effective theory is hence rather minimal. Unlike, for instance, the DSB models based on $SU(N_c)$ dynamics, it only contains meson fields and no other, more complicated composite states such as baryons and antibaryons. A further virtue of the IYIT model is its vector-like matter content, which facilitates its analysis and which makes it easier to embed it into more complete scenarios. Chiral models, such as those presented in Ref. [18], tend, by contrast, to be more involved and are perhaps less suited for further generalizations [21].

For our special choice of quark flavors, $N_f = N_c + 1$, no dynamical (ADS) superpotential [22] is generated at low energies. The quantum moduli space is instead simply spanned by the

⁵In the convention used here, the strongly coupled $SP(1)$ theory is equivalent to an $SU(2)$ gauge theory.

$N_f(2N_f - 1)$ flat meson directions M^{ij} , subject to the following constraint [23],

$$\text{Pf}(M^{ij}) = \Lambda^{N_c+1}, \quad (8)$$

where $\text{Pf}(M)$ denotes the Pfaffian of the antisymmetric meson matrix M , $[\text{Pf}(M)]^2 = \det(M)$. This constraint is the quantum mechanically deformed version of the classical moduli constraint, $\text{Pf}(M) = 0$, where the appearance of the dynamical scale on the right-hand side of Eq. (8) is due to nonperturbative instanton effects. A convenient way to implement the deformed moduli constraint when studying the quantum moduli space of the $SP(N_c)$ theory is to include it directly into the effective superpotential in the form of a Lagrange constraint term,

$$W_{\text{eff}} \propto \frac{T}{\Lambda^{N_c-1}} [\text{Pf}(M^{ij}) - \Lambda^{N_c+1}], \quad (9)$$

with the chiral superfield T representing a Lagrange multiplier. The overall normalization of this effective superpotential is unfortunately uncalculable as the nature of the Kähler potential for the field T is unknown. T certainly does not possess a perturbative Kähler potential and whether or not it possesses a nonperturbative Kähler potential is an open question. If strong-coupling effects below the dynamical scale should happen to generate a Kähler potential for T , the superpotential in Eq. (9) would end up having a definite normalization,

$$W_{\text{eff}} = \lambda_T \frac{T}{\Lambda^{N_c-1}} [\text{Pf}(M^{ij}) - \Lambda^{N_c+1}], \quad (10)$$

with T being canonically normalized and for some finite coupling constant λ_T . If, on the other hand, no Kähler potential should be generated, we would have to interpret T as a mere auxiliary field. This would then correspond to the limit $\lambda_T \rightarrow \infty$ in the above superpotential. As we are unable to calculate the Kähler potential for the field T , we will simply decouple all effects related to it in the following. Practically speaking, we will do so by assuming that the dimensionless parameter λ_T is much larger than all other coupling constants in the theory.

In the above outlined setup, dynamical SUSY breaking is now achieved by stabilizing all flat directions in moduli space by means of appropriate Yukawa interactions. For every flat direction M^{ij} , we introduce a chiral singlet field Z_{ij} , which we then couple to the fundamental quark fields in the tree-level superpotential as follows,⁶

$$W_{\text{tree}} = \frac{1}{2} \lambda_{kl}^{ij} Z_{ij} Q^k Q^l, \quad \lambda_{ij}^{kl} = -\lambda_{ji}^{kl} = -\lambda_{ij}^{lk}, \quad Z_{ij} = -Z_{ji}. \quad (11)$$

Here, we assume all complex phases of the $\mathcal{O}(1)$ Yukawa coupling constants to be absorbed in the singlet fields Z_{ij} for simplicity. Note that the maximal flavor symmetry of this tree-level superpotential corresponds to a global $SU(2N_f)$ symmetry, provided that all Yukawa couplings are equal, $\lambda_{kl}^{ij} \equiv \lambda$. We shall, however, only be interested in Abelian subgroups of this maximal flavor symmetry, which is why we are free to redefine the fields Z_{ij} such that Eq. (11) turns into

$$W_{\text{tree}} = \frac{1}{2} \lambda_{ij} Z_{ij} Q^i Q^j. \quad (12)$$

⁶Throughout this paper, the flavor indices i, j, k, l always run from 1 to $2N_f$.

At energies below the dynamical scale, this superpotential can be reformulated in terms of the meson fields M^{ij} , so that the full effective superpotential is eventually given as

$$W_{\text{eff}} \simeq \lambda_T \frac{T}{\Lambda^{N_c-1}} [\text{Pf}(M^{ij}) - \Lambda^{N_c+1}] + \frac{1}{2} \lambda_{ij} \Lambda Z_{ij} M^{ij}, \quad (13)$$

where we have neglected all corrections to the Yukawa couplings λ_{ij} that arise when running down from high to low energies. This means in particular that the meson fields in Eq. (13) are supposed to represent the canonically normalized DOFs at low energies. The crucial property of the superpotential in Eq. (13) is that it leads to F-term conditions that cannot all be satisfied simultaneously, as long as none of the Yukawa couplings λ_{ij} is actually zero. SUSY is therefore spontaneously broken via the O’Raifeartaigh mechanism. On the other hand, in the case of one of the couplings λ_{ij} being zero, the low-energy vacuum is located at infinity in moduli space and SUSY remains preserved. Likewise, for more than one Yukawa coupling being zero, we recur to the original situation, in which the moduli space exhibits a number of flat directions, along which SUSY is unbroken. In the following, we will disregard these possibilities and focus on the case of generic, nonzero Yukawa couplings, so that SUSY is always dynamically broken.

2.2 Effective FI-term upon weakly gauging a global $U(1)$ flavor symmetry

In the remainder of this section, we will now focus on the case of $SP(1) \cong SU(2)$ dynamics in combination with $N_f = 2$ quark flavors and illustrate how the IYIT model may allow for the dynamical generation of an effective FI-term. In doing so, we will also discuss the magnitude of the SUSY breaking scale and outline how the fundamental DOFs eventually end up being distributed in the low-energy effective theory. Here, we will in particular observe that the role played by some of the fundamental DOFs turns out to be slightly different than in the usual IYIT model without an Abelian FI-term. To start with, let us inspect once more the tree-level superpotential in Eq. (11) for the special case of $N_f = 2$. This superpotential exhibits an axial $U(1)$ symmetry associated with a Q^i phase rotation. It is this $U(1)$ symmetry for which we are now going to generate a nonvanishing effective D-term. In the first step, we first of all need to (weakly) gauge this symmetry and assign appropriate gauge charges to the chiral fields of our model. We assign $U(1)$ charges to the quark fields Q^i as follows,

$$[Q_1] = [Q_2] = +\frac{1}{2}, \quad [Q_3] = [Q_4] = -\frac{1}{2}. \quad (14)$$

Correspondingly, the six singlet fields Z_{ij} then carry the following charges,

$$[Z_{12}] = -1, \quad [Z_{34}] = +1, \quad [Z_{13}] = [Z_{14}] = [Z_{23}] = [Z_{24}] = 0. \quad (15)$$

For the ease of notation, we will therefore refer to Z_{12} as Z_- , to Z_{34} as Z_+ and to Z_{13} , Z_{14} , Z_{23} , and Z_{24} as Z_1^0 , Z_2^0 , Z_3^0 , and Z_4^0 in the following. We emphasize that our charge assignment in Eqs. (14) and (15) is such that $\sum_i q_i = \sum_i q_i^3 = 0$, as required so as to render the $U(1)$ flavor symmetry anomaly-free, cf. Eq. (3). Furthermore, also the mesons in the low-energy effective

Field	Q^1	Q^2	Q^3	Q^4	M_+	M_-	M_a^0	Z_+	Z_-	Z_a^0
$U(1)$ charge	$+1/2$	$+1/2$	$-1/2$	$-1/2$	$+1$	-1	0	$+1$	-1	0

Table 1: Chiral fields present in the minimal $SP(1)$ model (in the high-energy as well as in the low-energy regime) and charge assignment under the weakly gauged $U(1)$ flavor symmetry. Here, $a = 1, 2, 3, 4$. At low energies, the quark fields form mesons, of which in particular the charged mesons $M_+ = Q^1 Q^2 / \Lambda$ and $M_- = Q^3 Q^4 / \Lambda$ play an important role in the generation of the effective FI-term. All of the above charges could in principle also be rescaled, $q \rightarrow nq$, as long as the gauge coupling constant g is appropriately rescaled, too, $g \rightarrow g/n$.

theory now carry $U(1)$ gauge charges. According to their quark content, the six mesons M^{ij} are charged as follows,

$$[M^{12}] = +1, \quad [M^{34}] = -1, \quad [M^{13}] = [M^{14}] = [M^{23}] = [M^{24}] = 0. \quad (16)$$

Similarly as in the case of the singlet fields, we will from now on refer to M^{12} as M_+ , to M^{34} as M_- and to M^{13} , M^{14} , M^{23} , and M^{24} as M_1^0 , M_2^0 , M_3^0 , and M_4^0 . For an overview of our assignment of $U(1)$ gauge charges, cf. also Tab. 1.

In terms of the charge eigenstates at low energies, the effective superpotential in Eq. (13) can be rewritten as,

$$W_{\text{eff}} \simeq \lambda_T T [\text{Pf}(M^{ij}) - \Lambda^2] + \lambda_+ \Lambda M_+ Z_- + \lambda_- \Lambda M_- Z_+ + \lambda_a^0 \Lambda M_a^0 Z_a^0, \quad (17)$$

where $a = 1, 2, 3, 4$ and where we have renamed the Yukawa couplings λ_{ij} in Eq. (13) in an obvious way. Meanwhile, as we are dealing with the particular case of six mesons, the Pfaffian $\text{Pf}(M)$ of the antisymmetric meson matrix can be readily expanded in the following fashion,

$$\text{Pf}(M^{ij}) = M_+ M_- - M_1^0 M_4^0 + M_2^0 M_3^0. \quad (18)$$

Eq. (17) in combination with Eq. (18) allows to compute the F-term scalar potential for the scalar components of our meson and singlet fields. As we have gauged the global $U(1)$ flavor symmetry of the tree-level superpotential in Eq. (11), this potential now needs to be supplemented by a D-term scalar potential accounting for the $U(1)$ gauge interactions in the scalar sector,

$$V_D = \frac{g^2}{2} (|M_+|^2 - |M_-|^2 + |Z_+|^2 - |Z_-|^2 + \dots)^2, \quad (19)$$

where the ellipsis stands for hypothetical particles from other sectors which also carry $U(1)$ gauge charge. The central idea behind our mechanism for the dynamical generation of an effective D-term is now the following: By appropriately choosing the Yukawa couplings in Eq. (17) we are able to engineer the VEVs of the charged fields contributing to V_D in Eq. (19) in such a way that $\langle Z_{\pm} \rangle = 0$ and $\langle M_+ \rangle \neq \langle M_- \rangle$. This then results in a nonvanishing effective FI parameter⁷

$$\xi = \langle |M_-|^2 \rangle - \langle |M_+|^2 \rangle \neq 0. \quad (20)$$

⁷Unless stated otherwise, we shall always assume that except for M_+ and M_- no other charged field acquires a (large) VEV contributing to ξ . This means in particular that we shall take it for granted that the effective ξ parameter in Eq. (20) is not inadvertently absorbed by the VEV of another hidden-sector field.

Let us now calculate the VEVs of all scalar meson and singlet fields in our $SP(1)$ model. To facilitate our analysis, we assume a (slight) hierarchy between the Yukawa couplings for the uncharged fields and those for the charged fields, $\lambda_a^0 \gg \lambda_{\pm}$. This automatically guarantees that all neutral fields are stabilized around the origin, $\langle M_a^0 \rangle = \langle Z_a^0 \rangle = 0$. In consequence of that, the deformed moduli constraint turns into a condition for the charged mesons M_+ and M_- only,

$$\text{Pf}(M^{ij}) = \Lambda^2 \quad \rightarrow \quad M_+ M_- = \Lambda^2. \quad (21)$$

This constraint is invariant under $U(1)$ super-gauge transformations, $M_{\pm} \rightarrow M_{\pm} e^{\pm S}$, for some superfield-valued super-gauge transformation parameter S . A convenient way to parametrize the fluctuations of M_+ and M_- around their respective VEVs is hence the following,

$$M_+ = (\langle M_+ \rangle + M) e^S, \quad M_- = (\langle M_- \rangle + M) e^{-S}, \quad (22)$$

where M and S are chiral superfields of mass dimension 1 and 0, respectively. The advantage of this parametrization is that, later on, it will allow us to explicitly identify the super-gauge transformation parameter S with the Goldstone multiplet, which is absorbed by the $U(1)$ vector multiplet V upon the spontaneous breaking of the $U(1)$ symmetry, cf. Sec. 2.4. Plugging the expressions in Eq. (22) into the effective superpotential in Eq. (17) and setting all neutral meson and singlet fields to 0, we find

$$\begin{aligned} W_{\text{eff}} \simeq & \lambda_T T [\langle M_+ \rangle \langle M_- \rangle - \Lambda^2] + \lambda_T (\langle M_+ \rangle + \langle M_- \rangle) T M + \lambda_T T M^2 \\ & + \lambda_+ \Lambda (\langle M_+ \rangle + M) e^S Z_- + \lambda_- \Lambda (\langle M_- \rangle + M) e^{-S} Z_+. \end{aligned} \quad (23)$$

This form of the effective superpotential makes several important things immediately obvious: (i) The fields Z_+ and Z_- have both nonvanishing F-terms. They hence both contribute to the goldstino multiplet X responsible for the spontaneous breaking of SUSY. (ii) If the field T is indeed dynamical, i.e. if $\lambda_T \sim \mathcal{O}(1)$, it, too, possesses a nonvanishing F-term, rendering it also part of the goldstino multiplet. Here, an explicit calculation leads to $F_T \simeq \lambda_+ \lambda_- / \lambda_T \Lambda$. If, on the other hand, T is a mere auxiliary field, i.e. if $\lambda_T \rightarrow \infty$, its F-term vanishes and the deformed moduli constraint ends up being exactly fulfilled by the VEVs of the charged meson fields,

$$\lambda_T \rightarrow \infty, \quad F_T \rightarrow 0, \quad \langle M_+ \rangle \langle M_- \rangle - \Lambda^2 \rightarrow 0. \quad (24)$$

(iii) The fields T and M share a supersymmetric mass term. Once we require that the deformed moduli constraint be satisfied exactly, i.e. once we send λ_T to infinity, this mass blows up. The fields T and M thus become very heavy, which causes them to decouple from the low-energy dynamics. In this limit, the deformed moduli constraint then eliminates the (auxiliary) field T as well as the field M , i.e. one complete chiral multiplet of mesonic DOFs. As already mentioned below Eq. (10), we shall work in exactly this limit in the following. We reiterate once more that this amounts to considering the field T as a mere undynamical Lagrange multiplier.⁸

⁸From now on, we will therefore simply set $F_T = 0$. This certainly does not limit the validity of our construction, because, even in the case of a dynamical field T , the F-term F_T is typically subdominant. One can show that for not-too-small λ_T , i.e. as long as $\lambda_T^2 > 2\lambda_+\lambda_-$, it is in fact always smaller than the singlet F-terms, $|F_T| < |F_{Z_{\pm}}|$.

Setting M to its VEV, $\langle M \rangle = 0$, and using that $\langle M_+ \rangle \langle M_- \rangle = \Lambda^2$ for $\lambda_T \rightarrow \infty$, we now have

$$W_{\text{eff}} \simeq \lambda_+ \Lambda M_+ Z_- + \lambda_- \Lambda M_- Z_+, \quad (25)$$

where the charged meson fields M_+ and M_- are to be expanded around their VEVs as follows,

$$M_+ = \langle M_+ \rangle e^S, \quad M_- = \langle M_- \rangle e^{-S} = \frac{\Lambda^2}{\langle M_+ \rangle} e^{-S} = \frac{\Lambda^2}{M_+}. \quad (26)$$

This illustrates once more that the deformed moduli constraint is also satisfied on the level of the chiral superfields, $M_+ M_- = \Lambda^2$, and not only on the level of the scalar VEVs. In order to actually calculate $\langle M_+ \rangle$ and $\langle M_- \rangle$, we need to minimize the F-term scalar potential resulting from Eq. (25) in combination with the D-term scalar potential in Eq. (19), while taking into account that $M_- = \Lambda^2/M_+$ for all values of M_+ . Let us assume for a moment that $\langle Z_{\pm} \rangle = 0$ (we will justify this assumption further below in Sec. 2.4). We then find for the VEVs of the charged meson fields

$$\langle |M_{\pm}|^2 \rangle = \frac{\lambda_{\mp}}{\lambda_{\pm}} \Lambda^2 \left[1 + \frac{g^2}{2} \left(\frac{\lambda_{\pm}}{\lambda_{\mp}^3} - \frac{\lambda_{\mp}}{\lambda_{\pm}^3} \right) + \mathcal{O}(g^4) \right]. \quad (27)$$

As we are only interested in the limit of a weakly gauged $U(1)$ symmetry, $g \ll 1$, we can safely neglect all higher-order corrections in g . In the generic case, we expect the Yukawa couplings λ_+ and λ_- to (at least slightly) differ from each other, $\lambda_+ \neq \lambda_-$, so that the VEVs of the two mesons M_+ and M_- do not (exactly) coincide, $\langle M_+ \rangle \neq \langle M_- \rangle$. According to Eq. (20), this then induces the following effective FI parameter,

$$\xi = \Lambda^2 \left(\frac{\lambda_+}{\lambda_-} - \frac{\lambda_-}{\lambda_+} \right) \left[1 - \frac{g^2}{2} \frac{(\lambda_+^2 + \lambda_-^2)^2}{\lambda_+^3 \lambda_-^3} + \mathcal{O}(g^4) \right]. \quad (28)$$

This expression is the main result of our paper. For given values of the Yukawa coupling constants λ_+ and λ_- , it is largest in the limit $g \rightarrow 0$. This directly reflects the influence of the D-term scalar potential in Eq. (19) on the VEVs of the charged mesons. The D-term scalar potential drives these VEVs towards a common value, which results in a smaller ξ parameter, as soon as the D-term potential gains in importance. Moreover, we find that, for λ_+ and λ_- of $\mathcal{O}(1)$, the magnitude of the FI parameter is directly controlled by the dynamical scale, $\sqrt{|\xi|} \sim \Lambda$, as anticipated. At the same time, the VEV of the auxiliary D field is suppressed by the small $U(1)$ gauge coupling constant, $\langle D \rangle = g \xi \sim g \Lambda^2$. This needs to be compared with the magnitude of the total F-term resulting from the effective superpotential in Eq. (25),

$$\langle |F| \rangle \equiv \sqrt{\langle |F_{Z_+}|^2 \rangle + \langle |F_{Z_-}|^2 \rangle} = \exp \left[\frac{K_0}{2M_{\text{Pl}}^2} \right] \mu^2, \quad K_0 = \langle K \rangle, \quad \mu \equiv \sqrt[4]{2\lambda_+ \lambda_-} \Lambda, \quad (29)$$

which is typically of the order of the dynamical scale, $\langle |F| \rangle \sim \Lambda^2$. In the parameter region of interest, i.e. for a weakly gauged $U(1)$ symmetry and a not-too-strong hierarchy among the Yukawa coupling constants λ_+ and λ_- , the effective D-term is hence always smaller than the IYIT F-term, $\langle D \rangle < \langle |F| \rangle$, as expected, cf. also Eq. (5). Note, however, that in general the IYIT

model or extensions thereof may potentially also be able to accommodate much larger D-terms, $\langle D \rangle \gtrsim \langle |F| \rangle$, in case the $U(1)$ gauge coupling is taken to a larger value, $g \sim \lambda_{\pm}$. But, as our above derivation of the ξ parameter in Eq. (28) is only self-consistent under the assumption of a small gauge coupling constant, $g \ll \lambda_{\pm}$, we cannot make any further statement as to whether this is indeed the case or not. Instead, we leave a study of the IYIT model (or extensions thereof) in combination with a strongly gauged $U(1)$ flavor symmetry to future work and keep on focusing on the weakly gauged scenario in the following. Just as in our analysis so far, we can then continue to treat the $U(1)$ gauge interactions as a small perturbation to the Yukawa interactions encoded in the tree-level superpotential.

In summary, we conclude that the IYIT F-term $\langle |F| \rangle$, the FI parameter ξ as well as the effective D-term $\langle D \rangle$ are all related to the dynamical scale Λ of the strong $SP(1)$ gauge interactions,

$$\langle D \rangle / g \equiv \xi \sim \langle |F| \rangle \sim \Lambda^2. \quad (30)$$

For completeness, we also mention that the dynamical scale Λ derives in turn, via the effect of dimensional transmutation, from the RGE running of the strong gauge coupling constant g_s ,

$$\Lambda = M_{\text{Pl}} \exp \left[-\frac{8\pi^2}{b g_s(M_{\text{Pl}})} \right], \quad b = 3(N_c + 1) - N_f, \quad N_f = N_c + 1, \quad (31)$$

with b denoting the beta-function coefficient for the $SP(N_c)$ theory with N_f flavors and where we have assumed that the number of flavors does not change between the dynamical scale and the Planck scale. For $N_c = 1$, a strong gauge coupling constant of $g_s \simeq 2.0$ at the Planck scale then implies, for instance, a dynamical scale coinciding with the scale of grand unification,

$$N_c = 1, \quad g_s(M_{\text{Pl}}) \simeq 2.0, \quad \Lambda \simeq \Lambda_{\text{GUT}} \simeq 2.0 \times 10^{16} \text{ GeV}. \quad (32)$$

Likewise, varying the gauge coupling constant g_s between, say, 1 and 4π , we are able to generate values of the dynamical scale Λ ranging over almost nine order of magnitude,

$$N_c = 1, \quad g_s(M_{\text{Pl}}) \simeq 1..4\pi, \quad \Lambda \simeq 6.5 \times 10^9 \text{ GeV}..2.1 \times 10^{18} \text{ GeV}. \quad (33)$$

Together, Eqs. (30) and (33) thus illustrate explicitly that our dynamical mechanism for the generation of an effective FI-term is capable of yielding ξ values in a much larger range than the conventional string construction based on the Green-Schwarz mechanism. This is a major advantage of our dynamical, field theory-based scenario.

2.3 Consistent embedding into supergravity

In the above derivation of the effective FI parameter ξ , we solely worked in the limit of global SUSY and completely neglected all SUGRA effects. This immediately gives rise to two questions: (i) What are the quantitative changes in our result for the parameter ξ once we include higher-dimensional SUGRA corrections? And more importantly, (ii) are we at all allowed to couple our globally supersymmetric $SP(1)$ model to SUGRA without running into such conceptional

problems as we discussed them in the introduction? Assuming the answer to the second question is positive, the first question is easy to answer. Simply based on dimensional analysis, we expect all SUGRA corrections to our above expressions to be suppressed by at least one power of the ratio Λ/M_{Pl} . For not-too-large values of the dynamical scale, $\Lambda \lesssim 10^{-1} M_{\text{Pl}}$, all SUGRA corrections are therefore well under control. In order to answer the second question, it is sufficient to note that the superpotential as well as the Kähler potential of our original high-energy $SP(1)$ theory are $U(1)$ gauge-invariant by construction. The FZ-multiplet is therefore always well-defined along the entire RGE flow and we do not have to worry about any complications when coupling our model to SUGRA. However, in order to make the virtues of our dynamical mechanism more explicit, it turns out to be useful to examine the Kähler potential of the low-energy effective theory in a bit more detail. More precisely, we shall now identify and discuss the analog of the Kähler potential K_{FI} in Eq. (1) in our model.

The canonical Kähler potential for the meson fields M_{\pm} in global SUSY is given as follows,

$$K = M_+^\dagger e^{2gV} M_+ + M_-^\dagger e^{-2gV} M_- . \quad (34)$$

Parameterizing the fluctuations of M_+ and M_- around the low-energy vacuum as in Eq. (26), this Kähler potential can be written as

$$K = K_0 + 2g \left[\langle |M_+|^2 \rangle - \langle |M_-|^2 \rangle \right] \left[V + \frac{1}{2g} (S + S^\dagger) \right] + \dots , \quad (35)$$

where $K_0 = \langle |M_+|^2 \rangle + \langle |M_-|^2 \rangle$ and with the ellipsis denoting higher-order terms in the linear combination $V + (S + S^\dagger)/(2g)$. Of course, the prefactor of the above linear term is nothing but our effective FI parameter ξ , cf. Eq. (20). This leads us to identify K_{FI} in our model with

$$K_{\text{FI}} = -2g\xi \left[V + \frac{1}{2g} (S + S^\dagger) \right] , \quad (36)$$

which is of exactly the same form as the Kähler potential in Eq. (2) resulting from the Green-Schwarz mechanism in string theory. First of all, this illustrates that the field S indeed represents in fact nothing else than the Goldstone multiplet associated with the spontaneous breaking of the $U(1)$ symmetry, which renders the vector multiplet V massive via the (affine) Abelian Higgs mechanism, $V \rightarrow V_M = V + (S + S^\dagger)/(2g)$. But more than that, the similarity between our K_{FI} in Eq. (36) and the stringy Kähler potential in Eq. (2) also shows that we are able to consistently couple our effective FI-term to SUGRA for the same reason as in string theory—in contrast to the Kähler potential in Eq. (1), our Kähler potential K_{FI} in Eq. (36) does not violate the gauge invariance of the FZ-multiplet. As the field S shifts under super-gauge transformations, $S \rightarrow S - 2g\Psi$, it exactly compensates for the gauge shift in the vector field, $V \rightarrow V + \Psi + \Psi^\dagger$. This renders K_{FI} gauge-invariant, $K_{\text{FI}} \rightarrow K_{\text{FI}}$, which ultimately also preserves the gauge invariance of the FZ-multiplet. The advantage of our dynamically generated FI-term compared to its stringy counterpart, though, is that the modulus field, i.e. the real scalar DOF contained in $S + S^\dagger$, is always automatically stabilized by the SUSY-breaking F-terms arising in our strongly coupled

theory. Unlike in the case of the stringy modulus field Φ in Eq. (2), no extra mechanism is therefore required to stabilize S . In particular, thanks to the spontaneous breaking of SUSY, the mass of the field S always ends up being parametrically larger than the mass of the $U(1)$ vector boson A . At energies below the SUSY breaking scale, but above the vector boson mass, our effective FI-term hence manages to resemble a genuine, constant FI-term to very good approximation. Our dynamical mechanism is thus not only consistent in the context of SUGRA, but also appears to be very promising from a phenomenological point of view.

2.4 Mass spectrum and stabilization of the goldstino direction

To quantify the above statements about the mass of the field S and its relation to the vector boson mass, let us explicitly reformulate our $SP(1)$ SUSY breaking model in terms of the Goldstone field S and the singlet fields Z_+ and Z_- . This will also allow us to eventually prove that Z_+ and Z_- are indeed stabilized around their origin, cf. the comment above Eq. (27). Using Eqs. (25), (26) and (27), the effective superpotential may be rewritten as,⁹

$$W_{\text{eff}} \simeq \mu^2 [\cosh(S) X - \sinh(S) Y] , \quad \mu = \sqrt[4]{2\lambda_+\lambda_-} \Lambda , \quad (37)$$

where we have introduced the following two linear combinations of the fields Z_+ and Z_- ,

$$X = \frac{1}{\sqrt{2}} (Z_+ + Z_-) , \quad Y = \frac{1}{\sqrt{2}} (Z_+ - Z_-) . \quad (38)$$

As evident from Eq. (37), for a generic value of the Goldstone field S , both X and Y possess nonvanishing F-terms. The sum of the absolute values squared of these F-terms then yields the scalar potential for the complex scalar contained in S ,

$$V_S = |F_X|^2 + |F_Y|^2 = \mu^4 \cosh(\sqrt{2}c) , \quad S = \frac{1}{\sqrt{2}} (c + i\varphi) . \quad (39)$$

We hence find that c , the real component of the complex scalar in S , is stabilized at the origin, while φ , the imaginary component of the complex scalar in S , turns out to be a massless flat direction. This is consistent with the fact that φ is to be identified with the actual Goldstone phase that is absorbed by the $U(1)$ vector field A upon the spontaneous breaking of the $U(1)$ symmetry. Owing to ordinary gauge invariance, we are then always allowed to shift φ to zero, so that the entire S multiplet vanishes, $S = 0$. This gauge choice corresponds to unitary gauge, in which S disappears from the superpotential, because it is eaten by the vector multiplet V .

In unitary gauge, the superpotential then takes the following, particularly simple form,

$$W_{\text{eff}} \simeq \mu^2 X . \quad (40)$$

To study the particle spectrum of our $SP(1)$ theory in this gauge, one can no longer use the standard representation of the $U(1)$ vector multiplet that is commonly employed in Wess-Zumino

⁹Unless stated otherwise, we will neglect all effects due to the $U(1)$ gauge interactions in the following and simply work in zeroth order in the gauge coupling. Similarly as before, we will also neglect all SUGRA corrections.

gauge. Instead, one needs to know the actual Lagrangian of our Abelian Higgs model in unitary gauge, which includes, *inter alia*, all interactions of the fields X and Y with the massive vector multiplet V_M that are encoded in the Kähler potential. This Lagrangian has been derived and discussed in detail in Ref. [24]. Alternatively, we can, however, also simply keep the field S in the superpotential and perform all calculations in Wess-Zumino gauge. The conceptional difference between these two approaches then is that the Goldstone multiplet is regarded as a set of gauge DOFs in the former case, while it is regarded as a set of matter DOFs in the latter case. Of course, both approaches are guaranteed to lead to the same physical results by virtue of the Goldstone boson equivalence theorem. Based on the Lagrangian stated in Ref. [24], we have also confirmed the equivalence between both approaches by means of an explicit calculation.

For the purposes of the discussion in the present paper, we shall now continue our analysis in Wess-Zumino gauge. In doing so, we are free to focus, without loss of generality, on fluctuations of the Goldstone field around unitary gauge, $S = 0$. For S values close to 0, the superpotential in Eq. (37) can then be expanded in powers of S up to second order as follows

$$W_{\text{eff}} \simeq \mu^2 \left[X - S Y + \frac{1}{2} S^2 X + \mathcal{O}(S^3) \right]. \quad (41)$$

Likewise, we may expand the Kähler potential in Eq. (34) up to second order in S . Setting the vector field to its VEV, $\langle V \rangle = 0$, we find the following non-holomorphic terms in K ,

$$K \supset K_0 \left[1 + |S|^2 + \mathcal{O}(|S|^4) \right], \quad K_0 = \left(\frac{\lambda_-}{\lambda_+} + \frac{\lambda_+}{\lambda_-} \right) \Lambda^2. \quad (42)$$

The field S thus does not possess a canonical kinetic term. In order to canonically normalize it, we have to perform a field redefinition, $S \rightarrow K_0^{-1/2} S$, such that

$$\begin{aligned} K &\rightarrow K_0 + |S|^2 + \mathcal{O}(|S|^4), \\ W_{\text{eff}} &\rightarrow \mu^2 X - m S Y + \frac{m^2}{2\mu^2} S^2 X + \mathcal{O}(S^3), \end{aligned} \quad (43)$$

with the mass parameter m being defined as,

$$m = \frac{\mu^2}{K_0^{1/2}} = \lambda_h \Lambda, \quad \lambda_h = \left[\frac{1}{2} \left(\frac{1}{\lambda_+^2} + \frac{1}{\lambda_-^2} \right) \right]^{-1/2}, \quad (44)$$

where λ_h denotes the positive square root of the harmonic mean of λ_+^2 and λ_-^2 . Again, we can immediately infer several important results simply from the form of the superpotential.

(i) The O’Raifeartaigh-like structure of the superpotential is clearly evident. The field X eventually turns out to be the only field with a nonvanishing F -term.¹⁰ Hence, in unitary gauge, it directly corresponds to the goldstino multiplet associated with the spontaneous breaking of SUSY, cf. also Eq. (40). The complex scalar contained in X , the sgoldstino field, is typically

¹⁰According to Eq. (41), $\langle S^2 \rangle = -2$ appears, in fact, to represent a viable vacuum configuration as well. In this vacuum, F_X would then vanish and SUSY would instead be broken by the F -term belonging to the field Y . However, this is a fallacy, since Eq. (41) only holds in the limit of a small VEV for the Goldstone field, $S \ll 1$.

very light at tree level, which is why it represents what may be referred to as a pseudomodulus. The only contributions to its tree-level mass originate from higher-dimensional operators in the Kähler potential and are thus expected to be very small. At the same time, the fermionic component of X , the initially massless goldstino field \tilde{x} , is eaten by the gravitino upon the spontaneous breaking of SUSY, representing its longitudinal DOFs thereafter. Hence, the mass of the goldstino \tilde{x} eventually ends up corresponding to the gravitino mass $m_{3/2}$,

$$m_{\tilde{x}} \equiv m_{3/2} = \exp \left[\frac{K_0}{2M_{\text{Pl}}^2} \right] \frac{W_0}{M_{\text{Pl}}^2}, \quad W_0 = \langle W \rangle. \quad (45)$$

Here, W_0 denotes a constant in the superpotential that is generated in the course of spontaneous R symmetry breaking at high energies. It is tuned against the total SUSY breaking scale Λ_{SUSY} in the scalar potential, so as to realize an almost vanishing cosmological constant in the true vacuum.¹¹ This tuning implies the following phenomenological relation between $m_{3/2}$ and Λ_{SUSY} ,

$$m_{3/2} = \frac{\Lambda_{\text{SUSY}}^2}{\sqrt{3}M_{\text{Pl}}}, \quad \Lambda_{\text{SUSY}}^2 = \sqrt{\langle |F_{\text{tot}}|^2 \rangle + \frac{1}{2} \langle D^2 \rangle}. \quad (46)$$

Note that the above $|F_{\text{tot}}|$ does not necessarily need to coincide with the IYIT F-term $|F|$ in Eq. (29), since additional hidden sectors might still provide further sources of F-term-driven SUSY breaking. For our purposes, we can therefore treat the gravitino mass as a free parameter, which may or may not be determined by the dynamical scale in the strongly coupled sector.

For now, we would like to stress only two points: First, if there should be no further sources of SUSY breaking except for those coming from our strongly coupled sector, the SUSY breaking scale Λ_{SUSY} is also determined by the dynamical scale Λ . On the other hand, in presence of additional sources of SUSY breaking, Λ represents a lower bound for Λ_{SUSY} , cf. also Eq. (30),

$$\Lambda_{\text{SUSY}} \gtrsim \langle |F| \rangle \sim \xi \sim \Lambda, \quad m_{3/2} \gtrsim \frac{\Lambda^2}{\sqrt{3}M_{\text{Pl}}}. \quad (47)$$

Therefore, if we really aim at generating a ξ value of the order of Λ_{GUT} , we automatically let ourselves in for an extremely large SUSY breaking scale. In the case of such a large scale Λ_{SUSY} , all effects of SUSY decouple from low-energy physics and we could have no hope to solve any problems of the standard model by means of SUSY—this may or may not be regarded as a drawback. Alternatively, we may simply envision the dynamical generation of an effective ξ parameter of a much smaller magnitude. Then, also the SUSY breaking scale might end up lying in a phenomenologically more attractive range. Our second comment pertains the gravitino mass. Without any extra sources of SUSY breaking present, we know that $m_{3/2}$ takes a value of $\mathcal{O}(\Lambda^2/M_{\text{Pl}})$, cf. Eq. (46). All SUGRA corrections proportional to $m_{3/2}$ in our above analysis are then be suppressed by the ratio Λ/M_{Pl} . This is exactly what we already estimated simply based on dimensional analysis at the beginning of Sec. 2.3. For further comments on the mediation of SUSY breaking to the visible sector in our dynamical model, cf. Sec. 4.1.

¹¹In Ref. [21], we will present an extension of our general $SP(N_c)$ model, based on the idea of conformal SUSY breaking [25], that incorporates a dynamical explanation for this fine-tuning at least at the classical level.

(ii) The singlet field Y shares a large supersymmetric Dirac mass term with the Goldstone field S . Both fields are hence stabilized around the origin, $\langle Y \rangle = \langle S \rangle = 0$. Here, the fact that the linear combination $Y = (Z_+ - Z_-)/\sqrt{2}$ vanishes incidentally implies that the singlet fields Z_+ and Z_- are bound to take the same value in the true vacuum, $\langle Z_+ \rangle = \langle Z_- \rangle$, independently of the actual VEV of the linear combination $X = (Z_+ + Z_-)/\sqrt{2}$. Interestingly enough, this means that the contributions from the VEVs of the fields Z_+ and Z_- to the expectation value of the auxiliary D field, $\langle D \rangle \supset g (\langle |Z_-|^2 \rangle - \langle |Z_+|^2 \rangle)$, must necessarily cancel.

Upon closer inspection, we find that the two chiral fields S and Y give in fact rise to four real scalar as well as to two fermionic mass eigenstates,¹²

$$\{S, Y\}_{\text{bosonic DOFs}} \rightarrow \{c, \varphi, y_-, y_+\}, \quad \{S, Y\}_{\text{fermionic DOFs}} \rightarrow \{\tilde{s}, \tilde{y}\}. \quad (48)$$

Almost all of these fields possess an effective mass that depends on the complex scalar contained in the field X . Around the origin, i.e. at $\langle S \rangle = \langle Y \rangle = 0$, we find for the effective masses squared

$$\begin{aligned} m_c^2 &= m^2 \left[\frac{3}{2} + \frac{m^2}{2\mu^4} |X|^2 + \left(\frac{1}{4} + \frac{3m^2}{2\mu^4} |X|^2 + \frac{m^4}{4\mu^8} |X|^4 \right)^{1/2} \right], & m_\varphi^2 &= 0, \\ m_{y_-}^2 &= m^2 \left[\frac{3}{2} + \frac{m^2}{2\mu^4} |X|^2 - \left(\frac{1}{4} + \frac{3m^2}{2\mu^4} |X|^2 + \frac{m^4}{4\mu^8} |X|^4 \right)^{1/2} \right], & m_{y_+}^2 &= m^2 + \frac{m^4}{\mu^4} |X|^2, \\ m_{\tilde{s}}^2 &= m^2 \left[1 + \frac{m^2}{2\mu^4} |X|^2 + \left(\frac{m^2}{\mu^4} |X|^2 + \frac{m^4}{4\mu^8} |X|^4 \right)^{1/2} \right], \\ m_{\tilde{y}}^2 &= m^2 \left[1 + \frac{m^2}{2\mu^4} |X|^2 - \left(\frac{m^2}{\mu^4} |X|^2 + \frac{m^4}{4\mu^8} |X|^4 \right)^{1/2} \right]. \end{aligned} \quad (49)$$

Here, $m_{\tilde{s}}^2$ and $m_{\tilde{y}}^2$ are understood to denote the Majorana masses for the two Majorana fermions \tilde{s} and \tilde{y} at $X \neq 0$. On the other hand, at $X = 0$, the fermions \tilde{s} and \tilde{y} share a common mass m , so that they can be regarded as forming a Dirac fermion together. More generally speaking, evaluated at $X = 0$, the effective masses in Eq. (49) reduce to the following vacuum masses,

$$X = 0 : \quad m_c^2 = 2m^2, \quad m_\varphi^2 = 0, \quad m_{y_-}^2 = m_{y_+}^2 = m^2, \quad m_{\tilde{s}}^2 = m_{\tilde{y}}^2 = m^2. \quad (50)$$

As already mentioned below Eq. (39), the Goldstone phase φ turns out to be exactly massless and the real scalar c receives a large mass and is thus stabilized at the origin, $\langle c \rangle = 0$. Here, the scalar c corresponds to the real scalar in the massive vector multiplet $V_M \sim (c, \lambda, \tilde{s}, A)$ in unitary gauge. In addition, it plays the role of the real scalar modulus field in our construction of the effective FI-term in Eq. (36). As anticipated in Sec. 2.3, we now see explicitly that is indeed always automatically stabilized due to the F-term-driven SUSY breaking inherent in our

¹²The mixing between the DOFs contained in S and Y vanishes for $X = 0$. At $X = 0$, we then have $S \sim (c, \varphi, \tilde{s})$ and $Y \sim (y_-, y_+, \tilde{y})$. In the rigid limit, X vanishes in the true vacuum (as we will see shortly) and, hence, the role of the Goldstone multiplet is exclusively played by the field S . In SUGRA, however, $\langle X \rangle$ turns out to be of $\mathcal{O}(m_{3/2})$, so that the Goldstone DOFs end up corresponding to linear combinations of the DOFs in S and Y .

superpotential. In particular, for a weakly gauged $U(1)$ symmetry, its F-term-induced mass always easily satisfies the constraint formulated in the introduction, cf. Sec. 1.2,

$$m_c \sim \lambda_h \Lambda \gg g\sqrt{|\xi|} \sim g\Lambda, \quad (51)$$

which guarantees that the effective FI parameter ξ is not inadvertently absorbed by the VEV of the field c . Unlike in the case of string theory, we therefore do not have to invoke any separate mechanism to stabilize the modulus field. The dynamical stabilization of the modulus is already built into our model. Furthermore, the modulus mass is not directly tied to the gravitino mass, which leaves more freedom in the construction of realistic scenarios. In fact, barring additional sources of SUSY breaking, $m_{3/2}$ is typically much smaller than the modulus mass, $m_{3/2} \sim \Lambda^2/M_{\text{Pl}} \ll m_c$, cf. Eq. (46). These results regarding the scalar field c represent definite advantages of our dynamically generated FI-term over the stringy construction based on the Green-Schwarz mechanism.

The masses of the scalar c as well as of the Majorana fermion \tilde{s} now need to be compared with the masses of the vector boson A as well as of the gaugino λ . To compute these masses, we simply have to inspect the Kähler potential in Eq. (35), which can also be written as

$$K = K_0 - 2g\xi V_M + m_V^2 V_M^2 + \mathcal{O}(V_M^3), \quad V_M = V + \frac{1}{\sqrt{2}m_V}(S + S^\dagger), \quad m_V^2 = 2g^2 K_0. \quad (52)$$

As expected, we thus find that the vector boson A acquires a mass $m_A \equiv m_V \sim g\Lambda$. With respect to the gaugino λ , things are a little bit more complicated. Next to the mass for the vector boson A , the Kähler potential in Eq. (52) also induces a Dirac mass for the two fermions λ and \tilde{s} . Together with the Dirac mass m for \tilde{s} and \tilde{y} , cf. Eq. (50), and in the rigid limit, this then results in three mass eigenstates with the following tree-level masses in the true vacuum,

$$X = 0 : \quad m_\lambda^2 = 0, \quad m_{\tilde{s}}^2 = m_{\tilde{y}}^2 = m^2 + m_V^2 \simeq m^2. \quad (53)$$

At $X = 0$, the fermions \tilde{s} and \tilde{y} therefore form a Dirac fermion with mass m , while the gaugino λ turns out to be a Majorana fermion with vanishing tree-level mass. Here, the masslessness of the gaugino is a direct consequence of fermion number conservation: at $X = 0$, the Lagrangian only contains Dirac mass terms for the three Weyl fermions \tilde{y} , \tilde{s} and λ and no fermion number-violating Majorana mass terms. One of these three fermions therefore needs to end up being a massless Majorana fermion, which, in our case, is the gaugino λ . The origin of the fermion number conservation is, in turn, the continuous global R symmetry of the IYIT superpotential in the rigid limit, cf. Eq. (13), which forbids any Majorana mass terms in the fermionic Lagrangian.

This picture of the fermionic mass eigenstates, however, receives corrections from gravitational interactions. In the full, locally supersymmetric case, the value of the field X in the true vacuum does not vanish, but is rather of $\mathcal{O}(m_{3/2})$. This induces a mass splitting Δm^2 between the fermions \tilde{s} and \tilde{y} of $\mathcal{O}(\pm m m_{3/2})$, cf. Eq. (49), such that they end up representing independent Majorana fermions after all. At the same time, the gaugino λ and the fermion \tilde{y} acquire a common Dirac mass $m_{\lambda\tilde{y}} \sim g\langle X \rangle \sim g m_{3/2}$ in SUGRA via the supersymmetric

gaugino-fermion-scalar $U(1)$ gauge interactions. After diagonalizing the fermion mass matrix, this eventually gives rise to a Majorana mass $m_\lambda \sim m_V/m m_{\lambda\tilde{y}} \sim g^2/\lambda_h m_{3/2}$ for the gaugino mass eigenstate. Thus, the gaugino is, in fact, not exactly massless. In summary, we conclude that the spontaneous breaking of SUSY in our IYIT superpotential turns out to be responsible for a large mass splitting within the massive vector multiplet $V_M \sim (c, \lambda, \tilde{s}, A)$,

$$m_\lambda \sim \frac{g^2}{\lambda_h} m_{3/2}, \quad m_A \sim g \Lambda, \quad m_{\tilde{s}} \sim m_c \sim \lambda_h \Lambda, \quad m_\lambda \ll m_A \ll m_{\tilde{s}} \sim m_c. \quad (54)$$

In view of this result, two comments are in order. First, for a small gauge coupling constant g , the $U(1)$ gaugino λ turns out to be the lightest particle in the spectrum of our $SP(1)$ theory. In concrete phenomenological applications of our scenario, it might therefore be worthwhile to have a careful look at the role played by the gaugino. Under certain circumstances, it may perhaps play the role of dark matter; in other cases, its low mass may be used to place constraints on the dynamical scale Λ and/or the SUSY breaking scale Λ_{SUSY} . Second, at intermediate energies E in between the two mass scales m_V and m , the Goldstone multiplet S can be integrated out, while the gaugino and the vector boson are still light. At such energies, our effective FI-term then resembles a genuine, constant FI-term to very good approximation,

$$m_V \ll E \ll m : \quad K_{\text{FI}} \approx -2g \xi V. \quad (55)$$

After having succeeded in generating an effective FI parameter ξ by means of strong dynamics in Sec. 2.2, cf. Eq. (28), this observation is now the second main result of our paper. Remarkably enough, our dynamical FI-Term indeed bears the potential to imitate a constant FI-term.

(iii) We have already seen that the field Y is stabilized at zero. Hence, we have halfway proven that the VEVs of the fields Z_+ and Z_- indeed vanish. What remains to be done, though, is to show that, in the rigid limit, also the complex scalar contained in X , i.e. the sgoldstino, is stabilized at the origin. To do so, we first point out that the effective superpotential in Eq. (43) now also features a Yukawa interaction between the goldstino field X and the Goldstone field S that is quadratic in S . This Yukawa interaction induces an effective potential at one-loop that lifts the pseudoflat sgoldstino direction—the Goldstone multiplet associated with the spontaneous breaking of the $U(1)$ symmetry therefore stabilizes the sgoldstino direction associated with the spontaneous breaking of SUSY at the loop level. In order to explicitly calculate the sgoldstino mass m_X , we need to evaluate and differentiate the one-loop Coleman-Weinberg potential [26],

$$V_{\text{CW}} = \frac{1}{64\pi^2} \text{STr} \mathcal{M}^4 \ln \left[\frac{\mathcal{M}^2}{Q^2} \right], \quad m_X^2 = \left. \frac{\partial^2 V_{\text{CW}}}{\partial X \partial X^*} \right|_{X=0}. \quad (56)$$

Here, Q denotes an appropriate renormalization scale and \mathcal{M}^2 is the direct sum of the scalar and fermionic mass matrices squared. It contains in particular the six sgoldstino-dependent scalar and fermion masses in Eq. (49). Evaluating the Coleman-Weinberg potential by brute force, we

obtain the following positive sgoldstino mass squared at the origin,¹³

$$m_X^2 = \frac{2 \ln 2 - 1}{16\pi^2} \left(\frac{m}{\mu} \right)^4 m^2 = \frac{2 \ln 2 - 1}{32\pi^2} \left(\frac{\lambda_h}{\lambda_g} \right)^2 \lambda_h^4 \Lambda^2 = \frac{2 \ln 2 - 1}{4\pi^2} \frac{\lambda_+^5 \lambda_-^5}{(\lambda_+^2 + \lambda_-^2)^3} \Lambda^2, \quad (57)$$

where we have introduced λ_g to denote the positive square root of the geometric mean of λ_+^2 and λ_-^2 , cf. also Eq. (44). This result is independent of the renormalization scale Q and, more importantly, it is consistent with the expression found by the authors of Ref. [27] in the flavor-symmetric limit, in which all Yukawa couplings λ_{ij} are taken to be equal.¹⁴ To see this explicitly, note that in Ref. [27] the parameters m and μ are given as $m = \lambda \Lambda$ and $\mu = \lambda^{1/2} \Lambda$, with λ being the universal Yukawa coupling. Setting m and μ in Eq. (57) to these values, m_X^2 turns into

$$m_X^2 = \frac{2 \ln 2 - 1}{16\pi^2} \lambda^4 \Lambda^2. \quad (58)$$

This result is smaller than the corresponding expression in Ref. [27] by a factor of 5, since, in contrast to Ref. [27], we have initially decided, for simplicity, to decouple the four neutral meson-singlet pairs (M_a^0, Z_a^0) from the SUSY breaking dynamics of the charged fields (M_\pm, Z_\pm) . Moreover, the fact that we eventually find m_X^2 to be positive is not a coincidence. It is rather an implication of the global $U(1)_R$ symmetry of the superpotential in Eq. (43). This follows from a general theorem regarding the sgoldstino mass proven in Ref. [28], which states the following: Any SUSY breaking model of the O’Raifeartaigh type in which all chiral fields either carry R charge 0 or 2 is bound to lead to a positive mass squared for the pseudoflat direction. Given that the fields S , X and Y in Eq. (43) carry $U(1)_R$ charges 0, 2 and 2, respectively, this sufficient condition is evidently fulfilled in our case. The positive sign of m_X^2 in Eq. (57) is, hence, nothing but a consequence of the global $U(1)_R$ symmetry and our specific R charge assignment.

In addition to the loop-induced positive mass squared in Eq. (57), the sgoldstino mass also receives a further, uncalculable contribution from higher-dimensional operators in the Kähler potential that are induced by strong-coupling effects [27]. For not-too-large Yukawa couplings, $\lambda_\pm \ll 4\pi$, this contribution is, however, subdominant. Finally, we therefore conclude that the sgoldstino does not destabilize the vacuum. Instead, it is safely stabilized around the origin, $\langle X \rangle = 0$. This completes our proof that both singlet fields Z_+ and Z_- are indeed stabilized at the origin and justifies *a posteriori* our derivation of the effective FI parameter in Sec. 2.2.

3 Generalizations

3.1 Dynamical D-terms based on $SP(N_c)$ dynamics

While we have introduced the IYIT model of dynamical SUSY breaking for an arbitrary number of colors, $N_c \geq 1$, in Sec. 2.1, we have thereafter only focused on the minimal case of an $SP(1)$

¹³The first derivative of V_{CW} w.r.t. X vanishes. $X = 0$ is therefore indeed a local extremum of V_{CW} .

¹⁴A comparison in the case of different Yukawa couplings is not feasible, as the the corresponding expression in Ref. [27] unfortunately contains a few typos. It is, for instance, not of the correct mass dimension.

gauge theory in the remainder of Sec. 2. The generalization of our dynamical mechanism for the generation of an effective FI-term to larger numbers of colors is, however, straightforward. In this section, we will therefore only briefly summarize how the main results found in the previous chapter translate to the more general case of strongly coupled $SP(N_c)$ dynamics.

For a larger number of colors, $N_c \geq 2$, the tree-level superpotential of the IYIT model in Eq. (12) exhibits not only one, but several anomaly-free global $U(1)$ flavor symmetries. Each of these symmetries is equally suited to be used for the construction of a nonvanishing FI-term. After weakly gauging a particular $U(1)$ flavor symmetry of the tree-level superpotential, we then have to supplement the F-term potential with a corresponding $U(1)$ D-term potential. In the low-energy effective theory, this D-term potential takes the following form, cf. Eq. (19),

$$V_D = \frac{1}{2}D^2 = \frac{g^2}{2} \left[\sum_{a=1}^n q_a \left(|M_a|^2 - |Z_a|^2 \right) \right]^2, \quad \sum_{a=1}^n q_a = \sum_{a=1}^n q_a^3 = 0, \quad (59)$$

where we use $a = (i, j)$ as a collective index to label the $n = N_f(2N_f - 1)$ different pairs of meson-singlet charge eigenstates present at low energies. In order to calculate the expectation value of the auxiliary D field in the true vacuum, we again need to compute the VEVs of the meson and singlet fields. Just as in the minimal $SP(1)$ case, all singlet fields turn out to be stabilized at zero. This also includes the sgoldstino direction, which again receives a sufficiently large mass at the loop level. To facilitate our analysis of the meson VEVs, we again assume a particular hierarchy among the Yukawa coupling constants λ_{ij} , such that all mesons M^{ij} with $|i - j| \neq 1$ vanish in the true vacuum. This renders M^{ij} an antisymmetric tridiagonal matrix,

$$M^{ij} = M_\alpha J^{ij}, \quad \alpha = (2i - 1, 2i), \quad \alpha = 1, 2, \dots, N_f, \quad (60)$$

where the J^{ij} represent the entries of the symplectic form $J = \mathbb{1}_{N_f} \otimes i\sigma_2$. Here, $\mathbb{1}_{N_f}$ stands for the N_f -dimensional unit matrix and σ_2 is the second Pauli matrix. The form J is constructed such that it has unit Pfaffian, $\text{Pf}(J) = 1$. Making use of this ansatz, the F-term potential deriving from the superpotential in Eq. (13) is then minimized by the following nonzero VEVs,

$$\langle |M_\alpha|^2 \rangle^{1/2} = \frac{\lambda_g}{\lambda_\alpha} \Lambda + \mathcal{O}(g^2), \quad \lambda_g = \prod_{\alpha=1}^{N_f} \lambda_\alpha^{1/N_f}, \quad (61)$$

with λ_g denoting the geometric mean of the Yukawa couplings $\lambda_{12}, \lambda_{34}, \dots, \lambda_{2N_f-1, 2N_f}$. Also, note that now we are completely neglecting all corrections to the meson VEVs coming from the $U(1)$ gauge interactions. Inserting the meson VEVs in Eqs. (60) and (61) into the superpotential in Eq. (13), we are then able to identify the goldstino field X ,

$$M^{ij} \rightarrow \frac{\lambda_g}{\lambda_{ij}} \Lambda J^{ij}, \quad W_{\text{eff}} \rightarrow \mu^2 X, \quad \mu^2 = N_f^{1/2} \lambda_g \Lambda^2, \quad X = \frac{1}{N_f^{1/2}} \sum_{\alpha=1}^{N_f} Z_\alpha, \quad (62)$$

which generalizes the corresponding expressions found in the previous chapter, cf. Eqs. (37), (38) and (43). Similarly, plugging the meson VEVs into the D-term potential in Eq. (59), allows

us to deduce the generalized expression for the FI parameter ξ , cf. Eq. (28),

$$\xi = - \sum_{a=1}^n q_a \langle |M_a|^2 \rangle = -\lambda_g^2 \Lambda^2 \sum_{\alpha=1}^{N_f} \frac{q_\alpha}{\lambda_\alpha^2} + \mathcal{O}(g^2). \quad (63)$$

For generic Yukawa coupling constants λ_α , we hence find again a nonzero effective FI-term, the magnitude of which is determined by the dynamical scale, $\xi \sim \Lambda^2$. The corresponding effective D-term $\langle D \rangle$ is therefore again suppressed compared to the IYIT F-term $\langle |F| \rangle$, cf. Eq. (30),

$$\langle |F| \rangle \simeq \mu^2 \sim \Lambda^2 \gg \langle D \rangle \equiv g \xi \sim g \Lambda^2. \quad (64)$$

The fluctuations of the meson fields with nonzero VEV around the low-energy vacuum, cf. Eq. (61), can be parametrized in a similar way as in Eq. (22). Schematically, we have

$$M_\alpha = \mathcal{O}_{\alpha\beta} (S_1, S_2, \dots, S_{N_f-2}) \frac{1}{\lambda_\beta} [\lambda_g \Lambda + M] e^{q_\beta S_0}, \quad (65)$$

with M and S_0 denoting two chiral superfields of mass dimension 1 and 0, respectively, and where \mathcal{O} is an element of $SO(N_f, \mathbb{C}) / [SO(N_f - 1, \mathbb{C}) \times U(1)_\mathbb{C}]$, which is uniquely determined in terms of the $N_f - 2$ superfield-valued phases $S_1, S_2, \dots, S_{N_f-2}$. The field M couples again to the Lagrange multiplier T and hence decouples in the limit of a large Yukawa coupling λ_T , i.e. once we enforce the deformed moduli constraint to be fulfilled exactly. Meanwhile, the field S_0 can again be identified as the Goldstone multiplet that renders the $U(1)$ vector field V massive upon the spontaneous breaking of the $U(1)$ symmetry. From its interaction with the singlet fields Z_α in the superpotential, it acquires a supersymmetric Dirac mass m , which directly generalizes the mass parameter m introduced in Sec. 2.4, cf. Eq. (44),

$$m = \lambda_h \Lambda, \quad \lambda_h = \left[\frac{1}{N_f} \sum_{i=1}^{N_f} \frac{q_\alpha^2}{\lambda_\alpha^2} \right]^{-1/2}. \quad (66)$$

Hence, the real modulus field $c \in S_0$, which appears in our construction of the effective FI-term in the Kähler potential, cf. Eq. (36), is again parametrically heavier than the vector boson A ,

$$m_A^2 \equiv m_V^2 = 2g^2 \lambda_g^2 \Lambda^2 \sum_{i=1}^{N_f} \frac{q_\alpha^2}{\lambda_\alpha^2}. \quad (67)$$

At energies E in between the mass scales m and m_V , our effective FI-term in Eq. (63) therefore resembles once again a genuine, constant FI-term, cf. Eq. (55). In summary, we thus conclude that all of our main results derived in the special case of an $SP(1)$ theory readily carry over to the general scenario of strongly interacting $SP(N_c)$ dynamics. This illustrates that our dynamical mechanism for the generation of an effective FI-term, which we introduced in Sec. 2 only by reference to the simplest case of strongly coupled $SP(1)$ dynamics, does, in fact, not depend on any peculiarities of the IYIT model in its minimal realization. Instead, it is applicable in the full IYIT model for an arbitrary number of colors.

3.2 Dynamical D-terms based on $SU(N_c)$ dynamics

The results of the previous section immediately entail the question whether our dynamical mechanism could possibly also be implemented in other models of dynamical SUSY breaking. In this section, we shall briefly demonstrate that, properly taking into account gravitational corrections, it actually turns out to be impossible to dynamically generate an effective FI-term in the context of DSB models based on $SU(N_c)$ dynamics. This will help us formulate a general requirement pertaining the structure of candidate DSB models that needs to be satisfied, so that there is a chance of successfully accommodating an effective FI-term in the full SUGRA theory. The concrete investigation of further alternative DSB models is left for future work.

The generalization of the IYIT model to $SU(N_c)$ dynamics is based on supersymmetric QCD (SQCD) with $N_f = N_c$ flavors [29]. Here, we will only consider N_c values larger than 2, as the $SU(2)$ theory is equivalent to the $SP(1)$ theory discussed in Sec. 2. Now, every flavor is comprised of a pair of a chiral quark and a chiral antiquark field, Q^i and $\bar{Q}^{\bar{i}}$, which transform in the fundamental and antifundamental representations of $SU(N_c)$, respectively. The DOFs of the low-energy effective theory correspond to a set of $N_f^2 + 2$ gauge-invariant composite fields: N_f^2 mesons $M^{i\bar{j}}$ as well as a baryon B and an antibaryon \bar{B} ,

$$M^{i\bar{j}} = \frac{Q^i \bar{Q}^{\bar{j}}}{\Lambda}, \quad B = \epsilon_{i_1 i_2 \dots i_{N_c}} \frac{Q^{i_1} Q^{i_2} \dots Q^{i_{N_c}}}{\Lambda^{N_c-1}}, \quad \bar{B} = \epsilon_{\bar{i}_1 \bar{i}_2 \dots \bar{i}_{N_c}} \frac{\bar{Q}^{\bar{i}_1} \bar{Q}^{\bar{i}_2} \dots \bar{Q}^{\bar{i}_{N_c}}}{\Lambda^{N_c-1}}, \quad (68)$$

which are again subject to a quantum mechanically deformed moduli constraint [23],

$$B\bar{B} + \frac{\det(M^{i\bar{j}})}{\Lambda^{N_c-2}} = \Lambda^2. \quad (69)$$

In this theory, we can again spontaneously break SUSY via the O’Raifeartaigh mechanism by coupling the $2N_f$ fundamental high-energy DOFs, i.e. the quark and antiquark fields Q^i and $\bar{Q}^{\bar{i}}$, to $N_f^2 + 2$ singlet fields, $Z_{i\bar{j}}$, Z_0 and \bar{Z}_0 , in the tree-level superpotential. At low energies, this then results in the following effective superpotential of the O’Raifeartaigh type,

$$W_{\text{eff}} \simeq \lambda_T T \left[B\bar{B} + \frac{\det(M^{i\bar{j}})}{\Lambda^{N_c-2}} - \Lambda^2 \right] + \lambda_{i\bar{j}} \Lambda Z_{i\bar{j}} M^{i\bar{j}} + \kappa \left(\frac{\Lambda}{M_{\text{Pl}}} \right)^{N_c-2} \Lambda Z_0 B + \bar{\kappa} \left(\frac{\Lambda}{M_{\text{Pl}}} \right)^{N_c-2} \Lambda \bar{Z}_0 \bar{B}, \quad (70)$$

where $\lambda_{i\bar{j}}$, κ and $\bar{\kappa}$ denote dimensionless, nonzero Yukawa couplings of $\mathcal{O}(1)$, which we take to be real for simplicity. Likewise, λ_T denotes again the normalization of the Lagrange constraint term, by means of which we implement the deformed moduli constraint into the superpotential. Just as for our $SP(N_c)$ theories, we assume λ_T to be the largest coupling in the problem. In the low-energy vacuum of the F-term potential, all meson fields vanish and the deformed moduli constraint ends up being satisfied due to nonzero VEVs for the baryon and the antibaryon field,

$$M^{i\bar{j}} = 0, \quad \langle |B|^2 \rangle^{1/2} = \frac{\kappa_g}{\kappa} \Lambda, \quad \langle |\bar{B}|^2 \rangle^{1/2} = \frac{\kappa_g}{\bar{\kappa}} \Lambda, \quad \kappa_g = \sqrt{\kappa \bar{\kappa}}. \quad (71)$$

The SUSY breaking dynamics of the general $SU(N_c)$ theory are hence very similar to the corresponding dynamics of the $SP(1)$ model discussed in Sec. 2. Here, B and \bar{B} simply play the role of M_+ and M_- , while Z_0 and \bar{Z}_0 correspond to Z_+ and Z_- . If we now weakly gauge a $U(1)$ flavor symmetry of the effective superpotential, the VEVs of the composite states in the low-energy effective theory in Eq. (71) result in an effective FI parameter ξ of the following form,

$$\xi = -q_B \left[\langle |B|^2 \rangle - \langle |\bar{B}|^2 \rangle \right] = q_B \kappa_g^2 \Lambda^2 \left(\frac{1}{\bar{\kappa}^2} - \frac{1}{\kappa^2} \right), \quad (72)$$

where q_B is the charge of the baryon B under the weakly gauged $U(1)$ symmetry in question. Likewise, plugging the VEVs in Eq. (71) into Eq. (70), we find the effective superpotential of our $SU(N_c)$ theories in unitary gauge, which turns out to be very similar to the corresponding effective superpotential that we found in the $SP(1)$ case, cf. Eq. (40),

$$W_{\text{eff}} \simeq \mu^2 X, \quad X = \frac{1}{\sqrt{2}} (Z_0 + \bar{Z}_0), \quad \mu^2 = \sqrt{2\kappa\bar{\kappa}} \left(\frac{\Lambda}{M_{\text{Pl}}} \right)^{N_c-2} \Lambda^2. \quad (73)$$

The crucial difference w.r.t. the $SP(1)$ model is that, now, $\mu^2 \simeq \Lambda_{\text{SUSY}}^2$ is suppressed by at least one power of the ratio Λ/M_{Pl} compared to the dynamical scale squared. As we shall demonstrate in the following, this renders the effective FI parameter ξ in Eq. (72) inconsistent.

The main obstacle to the successful generation of an effective FI-term turns out to be the stabilization of the scalar component of the goldstino field, i.e. of the pseudomodulus X , taking into account gravitational corrections. To leading order in the inverse Planck mass, $1/M_{\text{Pl}}$, the one-loop corrected potential for the sgoldstino field X in SUGRA is given as

$$V_X^{\text{eff}} = \left(m_X^2 - 2m_{3/2}^2 \right) |X|^2 - 2m_{3/2} \mu^2 (X + X^*) + \mathcal{O}(M_{\text{Pl}}^{-2}), \quad (74)$$

where m_X denotes the effective sgoldstino mass induced at one loop. According to our analysis in Sec. 2.4 and given the expression for the SUSY breaking parameter μ in Eq. (73), it is now severely suppressed by a large power of the ratio Λ/M_{Pl} ,

$$m_X^2 \sim \frac{1}{16\pi^2} \left(\frac{m}{\mu} \right)^4 m^2 \sim \frac{\kappa^4}{16\pi^2} \left(\frac{\Lambda}{M_{\text{Pl}}} \right)^{4(N_c-2)} \Lambda^2. \quad (75)$$

At the same time, the gravitino mass $m_{3/2}$ is also affected by the suppression of the μ parameter,

$$m_{3/2}^2 \simeq \frac{\mu^4}{3M_{\text{Pl}}^2} \sim \kappa^2 \left(\frac{\Lambda}{M_{\text{Pl}}} \right)^{2(N_c-1)} \Lambda^2. \quad (76)$$

But for $N_c \geq 3$ colors, $m_{3/2}$ still ends up being larger than the loop-induced effective mass m_X ,

$$\frac{m_X}{m_{3/2}} \sim \frac{\kappa}{4\pi} \left(\frac{\Lambda}{M_{\text{Pl}}} \right)^{N_c-3}. \quad (77)$$

In all $SU(N_c)$ theories with $N_c \geq 3$ colors, the scalar potential for the sgoldstino field X is therefore negatively curved to leading order in $1/M_{\text{Pl}}$, cf. Eq. (74). This drives X to a large VEV around the Planck scale, $\langle |X| \rangle \sim M_{\text{Pl}}$, which eventually only becomes stabilized due to

higher-dimensional SUGRA corrections in the scalar potential. Such a large sgoldstino VEV is, however, fatal from the perspective of our dynamical mechanism for the generation of an effective FI-term as well as from the perspective of the entire $SU(N_c)$ DSB model.

First of all, it is inconsistent with our above derivation of the ξ parameter in Eq. (72), in which we assumed that $\langle X \rangle = \langle Z_0 \rangle = \langle \bar{Z}_0 \rangle = 0$. But more than that, the fact that field X is a linear combination of $U(1)$ charge eigenstates implies that, now, the $U(1)$ flavor symmetry is spontaneously broken at the Planck scale. The $U(1)$ vector field V thus acquires a mass of $\mathcal{O}(M_{\text{Pl}})$, such that it completely decouples from all low-energy physics. Our low-energy effective SUGRA theory therefore no longer features any $U(1)$ gauge interactions and there is no longer any D-term potential that could possibly contain an effective FI-term. Moreover, as far as the $SU(N_c)$ DSB model itself is concerned, we are now facing a twofold Polonyi problem [30]. During the stage of cosmic inflation, both the real and the imaginary part of the complex sgoldstino field are stabilized by means of a Hubble-induced mass at field values that tend to be a distance of $\mathcal{O}(M_{\text{Pl}})$ away from the true vacuum at $\langle |X| \rangle \sim M_{\text{Pl}}$. At the end of inflation, both real scalar fields hence begin to oscillate around the true vacuum with very large initial amplitudes. The subsequent decay of these oscillations then results in disastrous amounts of entropy production, which threatens the successful generation of the baryon asymmetry of the universe as well as primordial nucleosynthesis. To avoid such catastrophic consequences, the $SU(N)_c$ DSB model needs to be supplemented with a (dynamical) mechanism for the stabilization of the two real scalar DOFs contained in X . The minimal setup discussed above lacks such a mechanism.

In conclusion, we therefore find that our $SU(N_c)$ theories, while they represent viable models of dynamical SUSY breaking in the rigid limit, are not suited for the generation of an effective FI-term. In addition, the SUSY breaking dynamics themselves are endangered and serious cosmological problems arise once gravitational corrections are properly taken into account. The origin of all these problems can be traced back to the suppression of the SUSY breaking scale, $\mu \simeq \Lambda_{\text{SUSY}}$, in the superpotential. We expect that similar problems arise in every DSB model in which the nonzero F-terms responsible for SUSY breaking originate from higher-dimensional operators in the superpotential, i.e. in which the SUSY breaking scale is suppressed by powers of the ratio Λ/M_{Pl} . A natural possibility to avoid the resultant theoretical and phenomenological problems is then to focus on DSB models in which SUSY breaking is driven by relevant or marginal operators in the Lagrangian. Here, the $SP(N_c)$ models discussed in the present paper represent a prime example of DSB models that fulfill exactly this requirement.

4 Applications

In the two previous sections, we have presented and discussed in detail our dynamical mechanism for the generation of an effective FI-term in vector-like models of dynamical SUSY breaking. After having thus completed our field-theoretic analysis, we shall now briefly comment on possible phenomenological applications of our effective FI-terms in realistic models. Here, we shall

sketch in particular possible consequences of a nonvanishing FI-term for the mediation of SUSY breaking to the visible sector, cf. Sec. 4.1, as well as for cosmic inflation, cf. Sec. 4.2.

4.1 Mediation of supersymmetry breaking to the visible sector

Together with the choice of the mediation mechanism, the VEVs of the F- and D-terms set the scale of the soft masses in the low-energy effective theory. SUSY breaking will be mediated from the hidden to the visible sector at least by gravitational interactions. In this case, expanding the SUGRA scalar potential around the vacuum leads to soft masses for the MSSM scalars,¹⁵

$$\begin{aligned} V_F + V_D &= e^{K/M_{\text{Pl}}^2} \left(K^{i\bar{j}} D_i W D_{\bar{j}} \overline{W} - 3 \frac{|W|^2}{M_{\text{Pl}}^2} + e^{-K/M_{\text{Pl}}^2} \frac{1}{2} D^2 \right) \\ &= e^{K_0/M_{\text{Pl}}^2} \left(\frac{|W_0|^2}{M_{\text{Pl}}^4} |\phi_i|^2 - \frac{1}{2} \frac{\langle D^2 \rangle}{M_{\text{Pl}}^2} |\phi_i|^2 \right) + \mathcal{O}(|\phi_i|^4) + \dots, \end{aligned} \quad (78)$$

The first term in parenthesis is the familiar contribution from F-term SUSY breaking,

$$(m_0^F)^2 = e^{K_0/M_{\text{Pl}}^2} \frac{|W_0|^2}{M_{\text{Pl}}^4} = m_{3/2}^2 = \frac{\Lambda_{\text{SUSY}}^4}{3M_{\text{Pl}}^2}, \quad \Lambda_{\text{SUSY}}^4 = \langle |F_{\text{tot}}|^2 \rangle + \frac{1}{2} \langle D^2 \rangle, \quad (79)$$

whereas the second term yields tachyonic contributions to the scalar masses,

$$(m_0^D)^2 = -\frac{1}{2} \frac{\langle D^2 \rangle}{M_{\text{Pl}}^2} e^{K_0/M_{\text{Pl}}^2}. \quad (80)$$

Note that, in this setup, the MSSM scalars do not enter V_D . The mass contribution in Eq. (80) is rather a direct consequence of the requirement of a Minkowski vacuum, i.e. of requiring the parenthesis in the first line of Eq. (78) to vanish. However, contrary to the case of pure D-term mediation, these tachyonic contributions do not endanger the stability of the MSSM vacuum, since $\langle |F| \rangle > \langle D \rangle$ in the parameter range of interest. The MSSM gauginos remain massless at this order in $\langle |F| \rangle / M_{\text{Pl}}$, since the nonvanishing $U(1)$ gauge charges of Z_+ and Z_- do not allow a direct coupling between the goldstino field $X = (Z_+ + Z_-) / \sqrt{2}$ and the gauginos. Moreover, the R symmetry-conserving D-terms will not contribute to (R -violating) gaugino masses. In summary, in this minimal setup and in absence of any further SUSY breaking sector, both the MSSM scalars and the gravitino, cf. Eq. (46), obtain masses of $\mathcal{O}(\Lambda^2/M_{\text{Pl}})$. For a value of the dynamical scale Λ around the GUT scale, $\Lambda_{\text{GUT}} \sim 10^{16}$ GeV, this implies masses of about 10^{14} GeV and hence a complete decoupling from the standard model spectrum. This can, however, be avoided by simply lowering the value of the dynamical scale Λ (which translates into a smaller value of the strong coupling constant g_s at the Planck scale).

The soft masses discussed above can be enhanced by adding gauge mediation, i.e. by adding messenger particles which transform under $\mathcal{G}_{\text{GUT}} \supset \text{SM}$ and which couple to X and F_X (F-term gauge mediation) or are charged under the $U(1)$ symmetry (D-term gauge mediation). The former will yield positive contributions to the squared masses of the scalars, which are suppressed

¹⁵Here, following the notation of Ref. [31], $D_i W = \partial W / \partial \phi_i + M_{\text{Pl}}^{-2} W \partial K / \partial \phi_i$.

by the messenger scale instead of the Planck scale and which hence stabilize the scalars. As for the MSSM gauginos, only suppressed masses are induced by F-term gauge mediation [32]. Meanwhile, D-term gauge mediation yields scalar mass contributions of the following form [33],

$$m_0^{D, \text{GM}} \sim \frac{\alpha}{4\pi} \frac{\langle D^2 \rangle}{\Lambda^3} \sim \frac{\alpha g^2}{4\pi} \Lambda, \quad (81)$$

where α denotes the coupling strength of the standard model gauge group at the GUT scale. Unfortunately, the sign of this contribution is incalculable due to the strong dynamics involved in the hidden sector. Furthermore, R symmetry-breaking gaugino condensation in the strongly coupled sector can yield nonvanishing masses for the MSSM gauginos,

$$m_{1/2}^{D, \text{GM}} \sim \frac{\alpha}{4\pi} \frac{\langle D^4 \rangle}{\Lambda^7} \sim \frac{\alpha g^4}{4\pi} \Lambda. \quad (82)$$

For a further investigation on raising the gaugino masses through D-term gauge mediation, cf. also Ref. [34]. Finally, also when SUSY breaking is communicated to the visible sector through anomaly mediation, the additional D-term can help to stabilize the otherwise tachyonic scalars, if the MSSM fields carry suitable charges under the $U(1)$ symmetry [35].

In conclusion, we find that in phenomenologically viable setups the sparticle mass scale is controlled by the dynamical scale Λ , which is typically very large, cf. Eq. (33), far beyond the range of collider searches. In order to obtain a lighter sparticle spectrum, we would have to take the gauge coupling constant g to extremely small values, which may be less well motivated from a phenomenological point of view. On the other hand, g may be naturally suppressed for some specific physical reason. A more thorough investigation of this question is left for future work.

4.2 Cosmic inflation driven by a dynamical D-term

In the previous section, we investigated the effect of the dynamically generated FI-term on the mediation of SUSY breaking to the visible sector, assuming that both $\langle |F| \rangle$ and $\langle D \rangle$ remain unchanged until today. A further interesting situation arises when the D-term is responsible for a phase of cosmic inflation, ending once the D-term is absorbed by the VEV of another scalar field. Ensuring a (nearly) vanishing cosmological constant in the true vacuum then requires a cancellation among the contributions to the F-term potential, $m_{3/2}^2 = \langle |F_{\text{tot}}|^2 \rangle / (3M_{\text{Pl}}^2)$, cf. Eq. (46). Inflation is hence driven by the D-term potential only, even though $\langle D \rangle < \langle |F| \rangle$.

D-term hybrid inflation is an attractive, simple realization of cosmic inflation in SUSY [3]. The vacuum energy density driving inflation is provided by an (Abelian) FI-term. At the same time, the slope of the inflationary potential arises from radiative corrections due to perturbative Yukawa interactions of the inflaton field encoded in the tree-level superpotential,

$$W_{\text{inf}} = \gamma \Phi_0 \Phi_+ \Phi_- . \quad (83)$$

Here, Φ_0 contains the inflaton field ϕ and the Φ_{\pm} , carrying opposite charges under the $U(1)$ symmetry associated with the FI-term, contain the ‘waterfall’ field, which is responsible for ending inflation by absorbing the effective FI-term in its VEV. γ denotes a Yukawa coupling.

D-term hybrid inflation typically features super-Planckian field values. Hence, an embedding of the globally supersymmetric model into an effective SUGRA framework is mandatory. However, as discussed in Sec. 1, coupling a constant FI-term to SUGRA poses serious difficulties and the field-dependent FI-terms discussed in the literature so far are generically fixed to the Planck scale. This renders them useless for D-term hybrid inflation, where the measurement of the amplitude of the scalar power spectrum fixes the magnitude of the FI-term to a value close to the GUT scale. On the other hand, using the dynamically generated FI-term described in the present paper, the implementation of D-term hybrid inflation is straightforward. As long as $\langle \Phi_{\pm} \rangle = 0$, a large vacuum energy density, $V_0 = \frac{1}{2}g^2\xi^2$, is generated by the D-term, cf. Eq. (28). Once the waterfall field absorbs ξ in its VEV, the D-term scalar potential vanishes and a flat Minkowski vacuum (with SUSY broken by nonvanishing F-terms) is recovered. In this context, successful D-term hybrid inflation fixes the scale of SUSY breaking to $\Lambda_{\text{SUSY}} \gtrsim \sqrt{|\xi|} \sim \Lambda_{\text{GUT}}$.

This setup brings two important advantages over previous approaches. First, the FI-term can be easily generated at the GUT scale, as required by the data on the cosmic microwave background. Second, since the local $U(1)$ symmetry under which the Φ_{\pm} are charged is broken by the meson VEVs from the outset, no local cosmic strings are generated at the end of inflation. In general, we expect Φ_{\pm} to couple to other fields charged under the $U(1)$ symmetry via (higher-dimensional) operators which are $U(1)$ -invariant, but not invariant under Φ_{\pm} phase rotations, e.g. $K \supset \Phi_- \Phi_- Z_+ Z_+ / M_{\text{Pl}}^2$. In this case, the global symmetry associated with a rotation of the VEV of the waterfall field in the complex plane is explicitly broken, and no global cosmic strings are formed. If one forbids these terms by imposing an additional symmetry, then the phase transition ending inflation may, after all, produce topological defects associated with the breaking of just this symmetry. However, we stress that this is not the generic case and hence the production of cosmic strings at the end of inflation can be easily avoided. This extends the parameter space of D-term hybrid inflation to larger values of ξ and hence smaller values of the scalar spectral index n_s , i.e. into a phenomenologically interesting regime, which is otherwise ruled out by the observational bound on the tension of cosmic $U(1)$ strings [36].

A concrete realization of D-term hybrid inflation in SUGRA requires the choice of a suitable Kähler potential to resolve the eta problem. Here, an attractive possibility is to impose a shift symmetry along, say, the imaginary component $\text{Im}\{\phi\}$ of the inflaton field ϕ , so that the tree-level Kähler potential no longer depends on $\text{Im}\{\phi\}$, but only on the real component $\text{Re}\{\phi\}$,

$$\Phi_0 \rightarrow \Phi_0 + i\alpha M_{\text{Pl}}, \quad \alpha \in \mathbb{R} \quad \Rightarrow \quad K = \frac{1}{2}(\Phi_0 + \Phi_0^\dagger)^2 + \Phi_+^\dagger \Phi_+ + \Phi_-^\dagger \Phi_- . \quad (84)$$

Such a choice of the Kähler potential then allows for D-term inflation taking place in a chaotic regime, at field values high above the Planck scale, $\text{Im}\{\phi\} \gg M_{\text{Pl}}$. The large inflaton field excursion $\Delta\phi$ in this scenario then results in a large tensor-to-scalar ratio in accord with the large value recently claimed by the BICEP2 collaboration [37]. The idea of ‘chaotic D-term inflation’ has been proposed and investigated in Ref. [38] and recently been applied in Ref. [39]. Alternatively, one may assume a Kähler potential of the no-scale type, which leads to a small

tensor-to-scalar ratio in accord with the bound on this observable deduced from the PLANCK data [40]. Such a scenario has, for instance, been studied in Ref. [41].

5 Conclusions and outlook

The appearance of FI-terms in SUGRA is plagued by serious problems. On the one hand, fundamental, genuine FI-terms are very difficult, if not even impossible to realize in minimal SUGRA. Field-dependent FI-terms, on the other hand, can arise in the low-energy effective theory, e.g. via the Green-Schwarz mechanism in string theory. But their magnitude is generically found to be restricted to values around the Planck scale. In this paper, we have proposed a dynamical mechanism to generate a nonzero effective D-term which overcomes these limitations.

Our starting point is the IYIT model of dynamical SUSY breaking—a strongly coupled supersymmetric $SP(N_c)$ gauge theory with $N_f = N_c + 1$ flavors. In this DSB model, SUSY is spontaneously broken via the interplay of the deformed moduli constraint and tree-level Yukawa interactions that stabilize all flat directions in moduli space. Gauging an Abelian subgroup of the total flavor symmetry, the nonvanishing VEVs of the charged composite states at low energies then result (for generic values of the Yukawa couplings) in a nonzero effective D-term. This D-term can be interpreted as a field-dependent FI-term, the scale of which is determined by the dynamical scale Λ . As the scale Λ can be freely varied over many orders of magnitude, our dynamical mechanism hence allows for the generation of FI-terms at any scale between $\mathcal{O}(100)$ TeV and the Planck scale. Below the dynamical scale, but above the $U(1)$ vector boson mass scale (which is suppressed by the small $U(1)$ gauge coupling constant), the D-term acts as a genuine, constant FI-term. Moreover, the scalar modulus field ensuring the gauge invariance of the FI-term turns out to be automatically stabilized by the large IYIT F-term, rendering it parametrically heavier than the gravitino. The variable magnitude of our effective FI-term, the built-in dynamical stabilization of the modulus field and the large hierarchy between the SUSY breaking scale and the vector boson mass scale represent three major advantages of our mechanism over its stringy alternative based on the Green-Schwarz mechanism.

A minimal realization of dynamical mechanism is accomplished in the $SP(1)$ version of the IYIT model of dynamical SUSY breaking. This minimal scenario already suffices to illustrate all key features of our mechanism for the generation of an effective FI-term. It can be straightforwardly extended to the more general scenario of strongly interacting $SP(N_c)$ dynamics. On the other hand, DSB models in which the SUSY breaking scale is suppressed by some power of Λ/M_{Pl} turn out to be problematic: As we demonstrate for the case of $SU(N_c)$ dynamics, the $U(1)$ vector field then acquires a very large mass, so that it completely decouples from the low-energy physics. At sub-Planckian energies, there is hence no longer an auxiliary D field present in the theory, which precludes the possibility of having an effective FI-term. At the same time, the SUSY breaking dynamics themselves may be endangered, once SUGRA corrections are properly taken into account. This is an even more severe problem, which goes beyond

the question of whether or not it is possible to consistently generate an effective FI-term.

Dynamically generated D-terms open up a wide range of applications in SUSY model building. SUSY breaking via nonvanishing F-terms is inherent in this setup and its scale is set by the dynamical scale of the strong interactions; the gravitino mass is in particular given by $m_{3/2} \sim \Lambda^2/M_{\text{Pl}}$. In addition, nonvanishing D-terms contribute to the soft masses. In particular in gauge mediation, this yields phenomenologically interesting improvements over pure F-term SUSY breaking, as we have briefly reviewed in Sec. 4.1. Alternatively, if Λ is of order of the GUT scale, the dynamically generated D-term can be the source of cosmic inflation. Here, the simplest inflationary model featuring fields in the inflaton sector that are charged under the $U(1)$ symmetry, D-term hybrid inflation, gives results in accordance with current observations. Remarkably enough, since the $U(1)$ gauge symmetry is spontaneously broken from the outset, the cosmic string problem (which otherwise rules out the simplest model) is now absent.

Open questions remain. In this paper, we have focused on D-terms generated by gauging an Abelian flavor symmetry present in a certain class of vector-like DSB models. It remains to be investigated whether our mechanism can also be extended to non-Abelian flavor symmetries as well as to alternative DSB models. Furthermore, the mechanism presented here might be embedded into conformal SUSY breaking models, which would promise the possibility to combine a high SUSY breaking scale during inflation with a low SUSY breaking scale in the true vacuum [21]. Moreover, we expect further possible applications in the context of model building in supersymmetric gauge theories. For example, given the possibility of taking Λ to be of the order of the GUT scale, it would be interesting to identify the $U(1)$ as part of a GUT group. For example, an identification with $U(1)_{B-L}$, where $B-L$ denotes the difference between baryon number B and lepton number L , would allow to link the scales of cosmic inflation, leptogenesis and SUSY breaking in an intriguing way. However, in this case, the requirement that the effective FI-term not be absorbed in the VEV of any standard model field adds additional constraints and we leave a further investigation of this question to future work.

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